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THE ORIGIN AND INFLUENCE OF MAJOR TECHNICAL LIBRARIES IN PHILADELPHIA[1]

BY

JOHN R. SKEEN[2]

The four million volumes and 200 libraries of all types within the city limits of Philadelphia represent the acquisitions of two and a half centuries. These books give the record of the birth and development of intellectual life and accomplishment of the country more completely, perhaps, than can be found anywhere else. Some of the libraries filled the needs of their time and then ceased to grow; others continued to help succeeding generations. All except one were products of associations of men insistent upon acquiring knowledge and making it available to others. The history of these libraries becomes that of a few far-sighted and inquiring individuals, and the life and times in which they lived.

Three years ago there were 16 libraries in Philadelphia which were almost entirely chemical in nature. However, this subject is well represented in the general science group as well as in the medical collections and in those of the all-inclusive libraries of universities and colleges. The highest concentrations of chemical and closely allied works are to be found in the University of Pennsylvania with its Edgar Fahs Smith Memorial Collection in the Harrison Laboratory of Chemistry, its Towne Scientific School Library, and the various departmental collections; The Franklin Institute; the Philadelphia College of Pharmacy and Science; the Drexel Institute of Technology; and the College of Physicians. The Academy of Natural Sciences also is an important factor. These are the great growing collections of the city. The rest are of historic interest, for the general student, or are the relatively

[1] Presented at the 118th National Meeting of the American Chemical Society, Chicago, September 4–9, 1950, before the Division of Chemical Literature.

[2] Director of Market Research, Foster D. Snell, Inc., New York, N. Y.

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small, highly specialized type owned for the most part by firms producing chemicals and chemical products. The history of these libraries is the story of the emergence of chemistry from alchemy, the herbals, and the apothecary to the day when one speaks of an ink chemist and an adhesive chemist—in short, from philosophy to specialization to mass production. The record is of interest.

The first attempt to establish a public library in America is credited to James Logan. Private secretary and agent to William Penn, this bibliophile brought with him a large number of carefully selected books in 1699. Importations over the next half century brought the total to 6000. They were housed in a small building and, with proper guarantees, made available to the public. Logan bequeathed both books and building to the city and the former are now housed in the Ridgway Branch of the Free Library. This first great collection in the new land fostered "the spirit of the Renaissance and the tradition of Copernicus and Galileo." Although cryptic, the influence of Logan and his celebrated hobby was significant and lasting.

Shortly after, the individual mechanics library of the Carpenter's Company came into being by the necessity of the time. Philadelphia was little more than a minor and nondescript village about to become a boom town, and houses were urgently needed. To this end the Carpenter's Company of London was repeatedly solicited for craftsmen. This old guild had fallen upon evil days and was glad to oblige. As the great fire of 1666 destroyed most of the wooden structures of London the maximum of brick was used in rebuilding in the interest of safety. The masons replaced the carpenters and the guild was faced ". . . with the necessity of getting rid of a swarm of unhappy inmates." Many came to Philadelphia where, in 1724, they organized their Company with both philanthropic and trade union motives. A library was soon established, consisting of mechanical and architectural works imported mostly from London. The very secret "price books," the OPA regulations and specifications of the day, were issued in 1786 and subsequently for 50 years. Philadelphia rapidly became the center of architectural activity, with Carpenter's Hall the social center and general meeting place of the city. It still stands and houses the library of several thousand volumes. The inheritors of this type of activity in all its ramifications are The Franklin Institute and Drexel Institute of Technology.

The Junto was the second organization from which much developed. This was a literary and debating society started by Benjamin Franklin in 1727. In order to meet the consequent need for books, Franklin obtained 50 subscriptions at 40 shillings each and fixed annual dues at 10 more, thus initiating the first subscription library in 1731, with Louis Timothee as librarian. The interest of Logan was enlisted and he aided in the selection of purchases; George Whitefield sent books;

and John Wesley contributed his writings. For 34 years the collection was kept in the State House and then it was transferred to Carpenter's Hall. It is of some interest to note that Francis Hopkinson introduced the charging desk in 1764.

The American Philosophical Society was another result of Logan and the Junto. This was undoubtedly inspired by the movement in Europe to establish organizations for the pooling of ideas and knowledge relating to science and experimental philosophy. Such academies began in the Seventeenth Century in Italy and Germany. The Royal Society was chartered by Charles II only four years before the London fire, in which year the Academy of Sciences began in the France of Louis XIV. In 1743, Franklin succeeded in his plan for a general association of learned men extending throughout the colonies. Philadelphia was claimed to be the logical home of the Philosophical Society because of its "geographical location and the availability of Logan's library." A collection of books was quickly started and growth was stimulated by the appearance of the *Transactions* in 1771, eighteen copies of which were sent to the major universities of Europe with the object of receiving exchanges. Largely due to the prestige of Franklin this program met with every success, resulting in many sets of serial publications of foreign academies and societies. Again, because of the energy of Franklin, Philosophical Hall, located near the State House where the Society remains, was completed in 1789.

The second great patron of the Philosophical Society collection was Thomas Jefferson, who gave his "Draft of the Declaration of Independence" and "Notes on the State of Virginia," along with many books. Other items in the library worthy of special mention are Penn's Great Charter of 1701, vocabularies and grammars of Indian languages, the Michaus "Journal," the Lewis and Clark "Journals," and the Westover "Journal of the North Carolina Dispute." The pioneer work of William Maclure on geology appears in the *Transactions*. For a hundred years this activity continued, until the age of specialization gradually centered the attention of scientists on other technical organizations.

The beginnings of later organizations and libraries in Philadelphia may be attributed to its character in the mid-Eighteenth Century. It was the foremost city in America—with a population of 12,000 it was relatively large and most certainly rich. The reasons were several: William Penn granted the right of self-government to the colony; the "Liberty Bell" was bought to celebrate the Golden Jubilee; and the lure of inexpensive land so actively promoted by the proprietors was not to be denied. Then, too, the city was one of the few where members of the Church of England, the dissenters of all nations, and the Roman Catholics could and did live quietly together. The wealth was created by the land and a major port appeared. After 1730 the iron furnaces of the upper Schuylkill attained importance. A class consciousness

became manifest and a group of 65 families were the patricians, monopolizing all offices and directing most of the mercantile and cultural growth of the time. Travelers said that ". . . Philadelphia is the Athens of America and New York the Sparta." What of music, drama, science and literature were found in other cities at the end of the Century were present in Philadelphia 50 years before.

Education was casual and pretty much confined to the upper classes, with few schools. The William Penn Charter School, begun in 1711, was attended by 30 boys. Christ Church conducted one, as did a Swedish minister and the Moravians. Thomas Penn and Franklin early conceived the idea of starting a suitable academy (as did the Presbyterians), but nothing was achieved until the arrival of George Whitefield in 1739.

This founder of Calvinistic Methodism, "the greatest of all revivalists," was on a preaching mission. Co-incident was the "age of benevolence" in Europe characterized by the founding of schools for the education of the poor. This enthusiasm washed to the shores of Philadelphia, as Whitefield preached to multitudes. As a result, the mechanics, together with a few Moravians, planned a building and a free school. Whitefield Chapel was soon built and used for about everything except for what it was intended. After a decade, 50 citizens, almost entirely Anglicans, acquired the assets and the teaching of 145 boys soon began. The charity aspect was an afterthought and short-lived. Known successively as the Academy, and the College of Philadelphia, the non-sectarian University of Pennsylvania was thus actively begun. The library was started without delay and has long been the largest in the city.

Pious intent also caused the local Presbyterians to organize Princeton University, primarily to train ministers. It is doubtful whether either of these products of Philadelphia would have long survived without the lottery, since much of the money to start Princeton was raised by this means, and the college was similarly supported in significant part. Indeed, the custom was general. To build schools, churches, bridges, or anything else—hold a lottery and hold it if possible in Philadelphia. The practical patron of the most hallowed institutions may well be called Lady Luck.

The College had far-reaching effects upon the libraries of today through the establishment of the first medical school in America in 1765. John Morgan is recognized as the founder and the first six professors received their medical degrees from the University of Edinburgh. Together with Leyden, the most honored institutions of medicine, Edinburgh served as the prototype and dominant force for many years, and the new school made Philadelphia the medical center of America for a century and a half. Until his death Franklin was a most constructive and influential patron.

The new school owed much to the Pennsylvania Hospital, first of its kind in the colonies, and established by Dr. Thomas Bond in 1751. Franklin raised the money and served as president and in many other capacities as long as he lived. The staffs and teachers of both school and hospital were almost identical. Cooperation was complete and a relatively few men dominated this field in the city. Dr. John Fothergill, physician of London, may be said to have started the library. He sent one medical book 11 years after the hospital began. However, the immediate response with appropriations firmly established the first medical library in America. Now consisting of over 14,000 works, nearly all date from the first hundred years.

The desire to form a medical society was a natural outgrowth of the time. Morgan was one of the founders of the Philadelphia Medical Society in 1765. Perhaps this was the first colonial effort. In any event, Thomas Penn refused a charter and the young organization joined with the American Society for Promoting Useful Knowledge. This, in turn, merged with the older Philosophical Society. The idea of an association of colleagues modeled after the Royal College of Physicians in London persisted. Twenty-four years after the first attempt, the Philadelphia College of Physicians came into being. The two dozen founders were intimately connected with the medical school and the hospital. A common forum was established. The objects were many, and contributions to the public weal were almost innumerable. The library was among the first considerations and Morgan was a major help. The first volume of the *Transactions* was issued in 1793 and the series continues. With the development of specialization and the increasing activity of government in health, and food and drug administration, the public services of the College have been minor for a half century. It is now especially notable for its outstanding library of over 160,000 volumes.

By the end of the Eighteenth Century the effect of these institutions appeared in divers ways: chemistry assumed a definite character; the great Priestley arrived from England in 1794 and resided in Northumberland, Pa.; pharmacy was established and the first manufacture of chemicals began. Pharmacy and chemical manufacture were inseparable and the association began in the Hospital in 1752. Dr. Bond secured the appointment of Jonathan Roberts as the first apothecary in the city. His task, in the tradition of the English and French, was to make up medicines as instructed; however, Philadelphia was loath to adopt the practice and the physician continued to buy from the druggist, actually a warehouse keeper, and grind and mix them himself. Morgan attempted to separate the two functions without success. Finally, Dr. Abraham Chovet introduced the writing of prescriptions and, with the beginning of the first dispensary for the free distribution of medicines to the poor in 1786, the practice became general. The

apothecary then became a man of standing and there were 20 before the turn of the Century.

As a medical center, the city supported many druggists and they became the first to manufacture chemicals in this country. The firm of Christopher, Jr. and Charles Marshall were the pioneers with their muriate of ammonia and Glauber's salt. In 1793, John Harrison, educated in Europe and taught by Priestley, began the first production of sulfuric acid. The Wetherills made the first white lead sometime before 1790. All were Quakers.

Meantime, cultural and learned societies continued to appear. Early in the new Century the short-lived Botanical Society held in Philadelphia was founded. It is notable as an early expression of specialized interest and the publication of an essay by Benjamin Smith Barton, "Discourse on some Principal Desiderata in Natural History." In 1812 John Speakman, apothecary, suggested to Jacob Gilliams, dentist, that meetings would be helpful in the study of natural science. The first was held almost immediately and attended by Drs. Troost, pharmacist and chemist, and Camillus Mann, and a few others. Dr. Samuel Jackson of the University suggested the title, Academy of Natural Sciences. He declined to become a member, being of the opinion that this would be detrimental to his practice as a physician. Thomas Say and William Maclure joined the first year and the library was started. Say, the partner of Speakman, devoted all his energies to investigations, issuing six numbers of *American Conchology* and other papers. Maclure was largely responsible for the continued existence of the Academy by contributions of money and books, and it was he who urged the publication of the *Journal*. This appeared in 1817 and subsequently contained the contributions of all active naturalists of the period. Twenty-four years later the *Proceedings* were published. The Academy sponsored field trips, began an herbarium, zoological and mineral exhibits, and collected and preserved about every kind of natural specimen. With over 130,000 volumes the library is the largest of the kind in the city. The Philosophical Society then became less active in these fields.

The Athenaeum of Philadelphia was founded in 1814 as a library. The initiative was taken by 100 friends. William Tilghman, Chief Justice of Pennsylvania, was elected president; Dr. James Mease, lecturer in pharmacy at the University two years later, was vice president. Among other notables was Nicholas Biddle, then at the height of his power as president of the second United States Bank of doubtful fame. Peter S. Du Ponceau became the second president of the Athenaeum. The library was composed of works on law and literature, the journals of Congress, state papers, and scientific periodicals from home and abroad. This character is retained and volumes number over 50,000.

An effort at reform led to the beginning of the Philadelphia College

of Pharmacy and Science in 1821. Over 200 years before, James I had separated the apothecaries from the grocers by charter. The Pennsylvania Hospital appointed the first in the city. Its dominant position as a medical center seemed to encourage some startling abuses in the practice of medicine. The public was, of course, gullible. For instance, the dried juice of ripe pokeberries when made into a plaster was recommended for the treatment of cancer, catmint tea was used for colic, grapevine sap was sold as a hair restorer. Quacks flourished and many physicians and druggists lent their names to innumerable misrepresentations. Proprietary medicines and drugs imported from Europe were frequently fraudulent. Besides, there was little uniformity in the preparation of drugs and medicines and the training of apprentices was casual. To bring some order out of this situation, the University offered a course in pharmacy in 1816. Mease was the first lecturer, followed by Dr. John Redman Coxe. He consulted many in the trade and obtained consent to certificate all pharmacists for practice contingent upon being qualified by the faculty or taking the prescribed courses. However, all were not consulted and there was objection to what was regarded as high-handedness. A meeting was called at Carpenter's Hall by Henry Troth and Peter K. Lehman to decide upon a plan. Sixty-eight druggists and apothecaries agreed to form an association to reform and teach the science. Among them were Samuel Biddle, Samuel P. Wetherill and William Heyl. Charles Marshall was chosen president. Troost became the first professor of chemistry and Maclure lectured on mineralogy. The *American Journal of Pharmacy* appeared in 1825, the "Druggists Manual" the year after, and the library was started in 1822. It now has over 25,000 volumes relating to the allied sciences.

Specialization of interests and, thus, organizations continued as the industrial revolution got under way. Early in the Nineteenth Century, Dr. Birkbeck of Glasgow conceived the idea of lecturing to artisans on subjects which would aid them in the development of their callings. In 1823 the first Mechanics Institute began in London with Birkbeck in charge. The movement spread rapidly, but after the initial enthusiasm the artisans exhibited little interest. Since success depended upon attendance, the movement was short-lived. In Philadelphia, restriction of the membership to mechanics led to the formation of The Franklin Institute, in which membership was open to all.

Samuel V. Merrick at the age of 21 found himself "the owner of a workshop, without a mechanical education, with scarcely a mechanical idea." Because he lacked the necessary qualifications a local association of mechanics blackballed him. A friend, William Kneass, suggested that the young man start an organization suitable to his needs. There was little interest until Merrick was introduced to the newly elected "professor of Chemistry applied to Agriculture and Mechanic

Arts" at the University. Dr. William H. Keating already had tried to establish an institute of science and so two kindred spirits got to work. An open meeting was held at the State House and was largely attended. An election of officers followed within a fortnight and nearly 500 members were enrolled. The Franklin Institute came into existence February, 1824, with James Ronaldson, the leading type founder in the United States, as the first president. Among the members were John P. Wetherill, William Strickland, John Harrison, Clement C. Biddle.

The Institute was something new. It was devoted to the promotion of the mechanic arts and the dissemination of scientific knowledge. In this regard it paralleled the Pharmaceutical College but in a broader field—both put science to work. It contrasted with the Academy which adhered to "pure" science and the philosophy thereof and so was a specialized continuation of the Philosophical Society. The accomplishments of the Institute are legion, and its vitality and versatility are most impressive. The explanation in part lies in the character of the times and the city.

Philadelphia was the principal city in the United States and the largest. With a population of 137,000 it was larger than New York, with twice the number of Baltimore, and three times as many as Boston. The little oligarchy of cultured and wealthy men established over a century before continued. They not only made an Athens in America but were able to maintain it. The business of Government continued to be transacted in great part in the city. Many foreign ministers and consuls resided here instead of Washington. Here were the Mint and the second United States Bank. Here was the financial center. Wealth came from the farms and the port. The iron masters were established on the Schuylkill. Anthracite coal was just coming into use. This was the age of canal building: the Schuylkill Canal was just completed; the Union Canal connected Reading with the Susquehanna; the Juniata Canal was soon to provide another route to Pittsburgh; the Chesapeake and Delaware Canal made an inland waterway to Baltimore; and there were several others. The railroad era was dawning—three were built in Philadelphia within a decade after the Institute began.

Chemical enterprise, canals, mining, railroads all posed problems, looked for answers, stimulated invention, required patent protection. The Institute was ready and willing to perform these public services at the time they were urgently needed. It did a magnificent job. The old Hall on Seventh Street was designed by John Haviland, built in 1825, and occupied the next year. From the start it was too small. Lecture courses were given and a drawing or drafting school was begun. A high school was organized to teach ". . . English literature and ancient and modern languages." This was an expression of the free school for which Whitefield Chapel was built nearly a century before, and became the model of the Central High School when the city estab-

lished it shortly after. Interest in the industrial arts caused the Institute to found the School of Design for Women in 1850, the first of its kind. Aid and shelter were given the Pennsylvania Museum and School of Industrial Art in its early years.

Investigations embraced almost everything. The JOURNAL published on the strength of materials, the Institute began the first weather bureau in 1843, developed the standard screw thread twenty years later. The library was created almost immediately. The JOURNAL began in 1826 and opened the way for exchange relations with other societies, magazines and periodicals devoted to science and the useful arts. "From . . . this . . . has grown a reference library of scientific literature, in some branches unique, and, in extent and completeness, second to none in the United States. . . ."

Contributions to invention were invaluable. It will be remembered that letters patent were provided for in the Constitution (art. 8, sect. 1). These were granted by the Secretary of State until 1836. As might be expected it was costly and difficult to find out what had been invented. The Institute undertook to examine new machines and ideas and give advice and guidance to inventors, and these findings were published in the JOURNAL, edited by Dr. Thomas P. Jones. In 1828 he was appointed Superintendent of the Patent Office in Washington but continued editorship of the JOURNAL until his death 20 years later. The JOURNAL became the only published source of patent claims until 1842.

The old names continued. Alexander Dallas Bache, great-grandson of Franklin, presided over the Institute for 14 years. There is the list of Harrisons, Henry R. Heyl, George W. Farr—manufacturing chemists since 1818, the Lewises (also manufacturing chemists), John Maclure, the Wetherills, William Rush, Benjamin Say and the rest. The contributions of this group during the middle half of the last Century defy estimate. The library holds the original record of the industrial development of the day. At present numbering 135,000 volumes, the library is still vigorously growing.

The last great development in the technical field sprang from the plan of a financier. Philadelphia was long the financial center of America—Robert Morris, Hamilton and the first United States Bank, the Biddles, then the Drexels. Francis M. Drexel was an Austrian who arrived in Philadelphia in 1817. Starting as an artist in oils, he wandered around South America for a while and ultimately returned to open a brokerage house in 1838. The next year his son entered the business. Under Anthony J. Drexel, the firm became the foremost banking house of its day. It is of interest to note that Junius Morgan entered the same business only a few years before. The firm of Drexel, Morgan and Company was formed in 1871 in order to refund the government bond issues. Subsequent transactions caused the United States to become known as "the land of ten per cent" and the era of international finance began. Under J. Pierpont Morgan the Drexels lost

their dominant position and the financial center of the country moved to New York.

At the height of his influence, Drexel contemplated a school to teach and train young men to enter the industries of the city. The idea was to found a practical school where theory and application of industrial occupations would be presented. This contrasted with the University which remained professional in nature. To this end, therefore, organizations at home and abroad were visited and studied. Pratt Institute, perhaps, contributed most. The first building of Drexel Institute of Technology was dedicated in 1891, financed and endowed by the founder. The next year a library school was established following the pattern of the New York Library School which began five years before. Miss Alice B. Kroeger was the first librarian. With nearly 110,000 volumes this collection is now one of the largest devoted almost exclusively to science and engineering. As a school of mechanical design and technical training a close association with industry is maintained, particularly well adapted to an era of mass production.

Of the libraries in Philadelphia, about half include technical and scientific material. The location of any volume can readily be determined by consulting the Union Catalogue of the Philadelphia Metropolitan Area.

The history of the libraries of Philadelphia is in fact simply the record of accomplishment of able men stimulated by the common need and interest. The libraries are only means to ends peculiar to the times. Thus, from the Junto came the associations of colleagues, each more restricted than its predecessor: the American Philosophical Society, the College of Physicians, the Academy of Natural Science. Also from the Junto came indirectly the University of Pennsylvania with the medical school and the Philadelphia College of Pharmacy and Science. There is a trade guild sequence: the Carpenter's Company, The Franklin Institute, Drexel Institute of Technology. All exist today. The libraries of six contain most of the technical works of the city.

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CARRIER COMPENSATION FOR SERVOMECHANISMS[1]

BY

H. ELMORE BLANTON[2]

Part I

ABSTRACT

A carrier servo system is a servo system which employs carrier-frequency techniques for data transmission. The provision of compensation to improve the stability and the performance of such a system presents several unique problems. Theoretically, the compensation transfer characteristic should have an amplitude response which is symmetrical about the carrier frequency and a phase response which is skew-symmetrical about the carrier frequency, both considered relative to an arithmetical frequency scale. Transfer characteristics normally obtained from physically realizable structures possess the required symmetry if considered on a logarithmic frequency scale; therefore, they introduce distortion into the system. Accuracy of physically realizable transfer characteristics is limited by tolerances required on components of the compensation device and by the dependence of many of the devices on the absolute stability of the carrier frequency.

Carrier-compensation transfer characteristics may be realized through the use of passive electrical networks, electrical or electromechanical demodulation, compensation, and remodulation systems, feedback devices, and other means. Two main types of passive electrical networks have been employed, namely, those involving resistances, inductances, and capacitances, and those involving only resistances and capacitances. The former type of networks is the most general but is limited by the non-linearities introduced by non-ideal inductances. Parallel-T and bridge-T networks of the latter type of networks have found wide usage in carrier lead compensation. Electrical demodulation, compensation, and remodulation systems consist of the application of readily available electronic and non-carrier compensation techniques; however, electromechanical systems for obtaining similar results have been developed for carrier servo systems. The electrical systems do make available for carrier compensation a wide variety of compensation transfer characteristics, whereas electromechanical systems are more limited. Feedback devices are useful for obtaining time derivatives of the output. At least one method for minimizing the effect of carrier-frequency instability has been developed.

Passive electrical networks present perhaps the best method of obtaining carrier-compensation transfer characteristics; however, characteristics other than simple lead are normally difficult to obtain, and the use of non-carrier compensation techniques appears advisable. Carrier servo systems do have inherent advantages as compared with non-carrier systems and consequently require less compensation for acceptable response. Great advancement may be expected in the field of carrier compensation during the coming years.

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[1] Submitted in partial fulfillment of the requirements for the Electrical Engineering Seminar, Massachusetts Institute of Technology, 1949.

[2] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

1. GENERAL CONSIDERATIONS—INTRODUCTION

Although the field of servomechanisms is perhaps a quarter of a century old,[3] very rapid growth has been experienced during the recent years because of the demand for automatically controlled devices during and since World War II. This rapid growth has resulted in the lack of definite terminology, and coordination within the field, and the security classification associated with many of the projects has prevented the publication of much of the information on the types of circuits employed in the solution of different design problems; therefore, the necessity arises that the types of systems pertinent to this paper be defined and that a statement be made that the information herein presented is by no means an exhaustive treatment of the subject of carrier compensation for servomechanisms. Reference is made in many cases to work on this problem at the Massachusetts Institute of Technology, not because the Institute has made the largest contribution to the solution of the problem, although such may be the case, but because, in general, literature was not available to the author on work completed elsewhere.

In the current chapter a servo system will be defined, the uses of compensation in such a system will be discussed, and the distinction between non-carrier and carrier systems will be outlined.

1.1. Description of a Servo System

A servo system, or a servomechanism, has been defined in various ways by different authors. The majority of them will agree that it is a closed-loop control system, but with such a statement as a point of departure they contend that various and sundry devices may or may not be included under the general classification. Although an argument on this matter is beyond the scope of this paper, and even an exact definition of the term is not pertinent, a definition will be given in the interest of having a definite basis on which to build the discussion of carrier compensation. On the other hand, the material presented on carrier compensation may be applicable in systems other than those covered by the following definition and description of a servo system.

The most recently published text book on the subject of servomechanisms, and incidentally one of the most complete text books in that field, is *Principles of Servomechanisms* by Professors Brown and Campbell of the Massachusetts Institute of Technology. The authors have sought to establish an accurate definition for a servo system, and in so doing they also have greatly limited the types of devices which may be properly classed as servomechanisms. Their definition is a very

[3] One of the first articles which attempted to formulate the basis for this field was "Theory of Servomechanisms" by Professor H. L. Hazen of the Massachusetts Institute of Technology. It was published in the September, 1934 issue of this JOURNAL (Vol. 218, pp. 279–331).

wise one and will be used as a basis for the description of a servo system which is included in the ensuing paragraphs.

Professors Brown and Campbell state that "the servomechanism is an error-sensitive, follow-up, amplifying system permitting wide range of the input command remotely located from the element being controlled" (1, p. 5).[4] In accordance with this statement the following distinct features of a servo system may be outlined:

1. The system is actuated by a quantity which is a function of the response of the controlled operation as well as the command signal. Because of the existence of this feedback from system output to input, the system is described as a closed-loop or follow-up system.

2. The system will respond to an input or command signal which may vary continuously over a wide range of values.

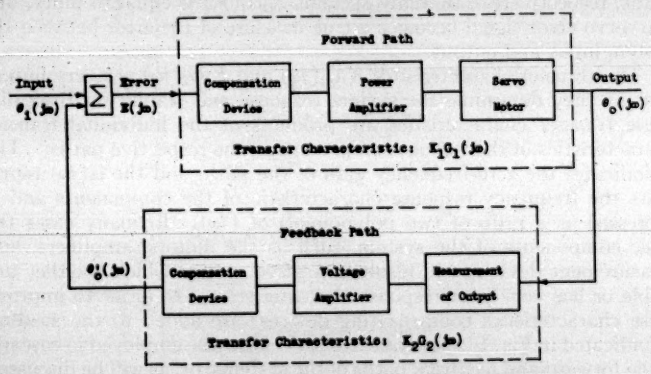

FIG. 1. Block diagram of a representative type of servomechanism.

3. The system may be controlled from a remote location and by a signal whose power level is much less than the power required by the controlled operation.

Added to these features is one dynamic feature which is necessary in a servo system, that is, the system must be designed so that the actuating signal will develop a rate of change of the response of the controlled operation rather than just a change in the response.

Figure 1 illustrates a representative type of servomechanism. Here the command signal and the response of the controlled operation are represented in the conventional manner by θ_i and θ_o, respectively. θ_i and θ_o may be considered as functions of time, or by appropriate

[4] The boldface numbers in parentheses refer to the references appended to Part II of this paper, to be published in the December 1950 issue.

transformation they may be represented as functions of the frequency variable $(j\omega)$. The latter form presents a basis for sinusoidal system analysis and is very useful. The system input and output are related by the following equations:

$$\theta_o(j\omega) = K_1G_1(j\omega) \times E(j\omega)$$

$$\theta_o'(j\omega) = K_2G_2(j\omega) \times \theta_o(j\omega)$$

$$E(j\omega) = \theta_i(j\omega) - \theta_o'(j\omega)$$

$$\theta_o(j\omega) = \frac{K_1G_1(j\omega)}{1 + K_1G_1(j\omega) \times K_2G_2(j\omega)}\,\theta_i(j\omega).$$

In these equations $E(j\omega)$ is the servo error signal, and $K_1G_1(j\omega)$ and $K_2G_2(j\omega)$ are the transfer characteristics of the forward and feedback paths, respectively. In many systems $K_2G_2(j\omega)$ is equal to unity, and the servo error signal becomes a true measure of the error between the system input and output.

The transfer characteristics $K_1G_1(j\omega)$ and $K_2G_2(j\omega)$ are very important as they determine the system response and stability. Basically, these transfer characteristics are products of the individual transfer characteristics of the various components in the respective paths. The K indicates the zero-frequency gain of the path, and the $G(j\omega)$ represents the frequency response characteristic of the components and is expressed as a ratio of two polynomials of $(j\omega)$. In many cases the basic components of the system, such as the motors, amplifiers, and measurement devices, will result in a servo system which is either unstable or has very poor response characteristics. In order to improve these characteristics compensating devices[5] are added to the system. As indicated in Fig. 1, compensation devices may be employed in cascade in the forward and feedback paths of the systems, or, as will be discussed later, separate devices may be used to measure the output of the system and additional feedback paths established parallel to the feedback path which is illustrated.

1.2. Uses of Compensation

Compensation devices are employed in connection with servo systems for one basic reason, that is, to improve the system response. For convenience in this discussion, the type of improvement obtained by the various compensation functions will be outlined, but no reference will be made at this time to the manner in which these compensation functions or transfer characteristics are obtained. The following remarks will be based on the sinusoidal method of servo analysis.

[5] In this paper "compensation devices" will be used as a general term to include electrical networks, electromechanical units, and any other means employed to obtain compensation transfer characteristics in servo systems.

One of the first reasons for attempting to use compensation in a servo system is to stabilize an otherwise unstable system, with stability here referring to the obtaining of a non-oscillatory system. As stability in this sense also may be obtained, in most cases, by the reduction of system loop gain, the problem may be classified as the improvement of

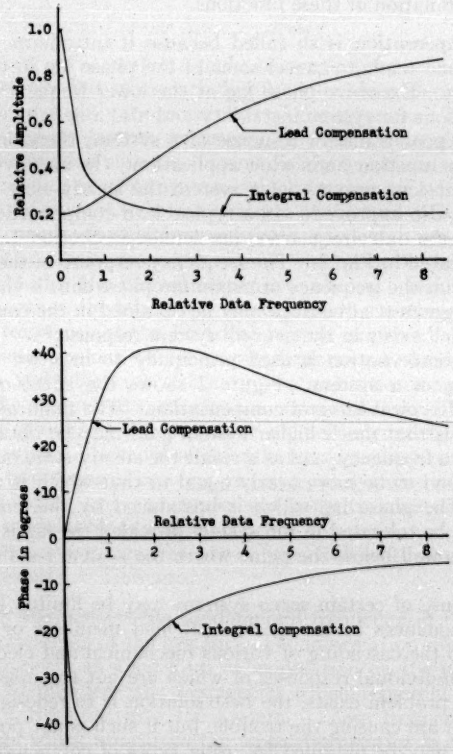

FIG. 2. Typical compensation functions.

the response of a system with very poor response. Improvement of the response of any system is achieved by:

1. Improving the steady-state response of the system.
2. Improving the transient response of the system.
3. Eliminating the effects of undesirable resonances and other irregularities in the response.

Methods of obtaining the desired improvement include the use of the following types of compensation functions:

1. Lead compensation.
2. Integral compensation.
3. Anti-resonant compensation.
4. A combination of these functions.

Lead compensation is so called because it introduces phase lead. This phase lead tends to cancel some of the phase lag in the response of the system. Excessive phase lag at the lower frequencies is one of the main reasons for system instability and also a definite contributing factor in the poor transient response of a system; therefore, the lead-compensation function finds wide application. In improving the stability and transient response of a system the steady-state response of the system is also improved. In a typical lead-compensation function, the effect on the data frequencies (as defined in Section 1.3) is similar to that illustrated in Fig. 2. The frequency response of the function is so adjusted that the frequency of maximum phase shift is the frequency at which the greatest advantage may be obtained in the cancellation of phase lag which exists in the over-all system response.[6]

Integral compensation is used principally to improve the steady-state response of a system. Figure 2 shows the effect on the data frequencies of typical integral compensation. The main advantage of this function is that much higher system gain may be obtained in the vicinity of zero frequency, and as a result the steady-state output of the system will tend to be more nearly equal to that which is required by the input. The phase lag which is introduced by this compensation function may be tolerated in the system provided the lag is introduced at frequencies well below the point where the system's stability would be affected.

The response of certain servo systems may be limited by the low-frequency resonances in the mechanical load members or by irregularities due to the cascading of various mechanical and electrical components, the individual responses of which are not the most desirable. When such a problem exists, the best solution is to redesign the components which are causing the trouble, but if such is not possible some improvement may be obtained by using types of compensation which are more complex than the previously mentioned lead-compensation and integral-compensation functions. Functions for this type of compensation include those which tend to cancel the effects of the undesirable resonances or which tend to cancel the frequency characteristic of the response of some component in the system, normally at the expense of

[6] A discussion of the adjustment of compensation to improve the response of a servo system is beyond the scope of this paper. The reader is referred to the treatment of this subject by Brown and Campbell (1) or by Hall (2).

shifting that frequency characteristic, perhaps in a modified form, to a region of higher frequency.

A specific problem in which electrical compensation can be used successfully to overcome undesirable characteristics of mechanical equipment is the stabilization of two-phase servo motors. This problem has been considered by a number of authors (3, 4, 5).

1.3. Non-Carrier and Carrier Servo Systems

Conceivably, servo systems could be synthesized utilizing no electrical components, but in essentially all applications electrical components are employed in various portions of the systems. The electrical components include motors, measuring devices, electronic amplifiers, and data transmission systems. Immediately questions arise concerning the advisability of using systems which employ only direct current, only alternating current, or a combination of the two. Experience has shown that certain applications require a particular type of system, whereas in other applications the system which can be the most easily synthesized to give the desired response is chosen. Before discussing this matter further, the types of systems should be distinguished on the basis of the frequency of the electrical current employed.

Servo systems which utilize carrier-frequency techniques for data transmission and which use alternating-current measuring devices and motors will be designated carrier or a-c. servo systems. Those systems which utilize principally direct current will be designated non-carrier or d-c. servo systems. Because of the relative advantages and disadvantages of the two types of systems, many servo systems employ a combination of carrier and non-carrier components interconnected with appropriate modulators and demodulators.

In a carrier servo system there are several different frequencies which must be considered. They are:

1. Carrier frequency—the fundamental frequency of the signal exciting all components of the system. The information introduced into the system appears as some form of modulation on the carrier signal.
2. Data frequencies—the frequencies which represent the data or information introduced into the system. These frequencies are the same as those which appear as the signal frequencies in a non-carrier system.
3. Signal frequencies—the frequencies included in the signal resulting from the modulation of the carrier frequency by the data frequencies.

Carrier frequencies of 60 and 400 cycles per second have been widely used in carrier servo systems. The former frequency is the standard power frequency, and the latter frequency is a compromise in the desire for a carrier frequency higher than 60 cycles per second and at the same

time one which can be easily obtained with electromechanical generating equipment. A 400-cycle carrier frequency must often be used instead of a 60-cycle carrier frequency in order to provide sufficient bandwidth to accommodate the higher data frequencies, as in a carrier servo system the highest data frequencies must be of the order of approximately twenty per cent of the carrier frequency.

The majority of the carrier servo systems in use today employ suppressed-carrier modulation. This type of amplitude modulation is well suited for use in servo systems as in balanced, steady-state conditions all signals tend to be zero, and dynamically the magnitude and the phase of the signals are functions of the magnitude and phase of the data frequencies. Suppressed-carrier modulated signals can be obtained easily from and can be employed with many types of measurement and control instruments, such as synchros, induction generators, accelerometers, and induction motors. A suppressed-carrier modulated signal is illustrated in Fig. 3.

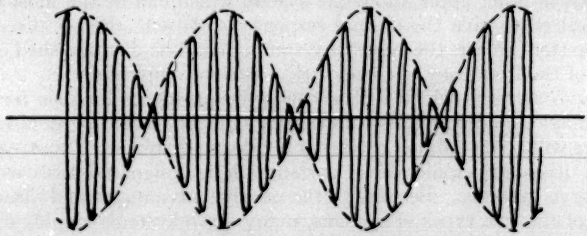

FIG. 3. Suppressed-carrier modulated signal.

2. REQUIREMENTS FOR CARRIER-COMPENSATION DEVICES

Very little consideration has been given in literature to the problem of providing compensation in a carrier servo system. Essentially all graphical and mathematical design techniques are based on the study of non-carrier systems, and the provision of compensation in non-carrier systems is so much easier and more accurate than in carrier systems that often the servo designers resort to non-carrier components in certain sections of carrier servo systems for the purpose of introducing the required compensation. This chapter will attempt to analyze the problem of carrier compensation by showing the type of compensation transfer characteristics which are necessary and some of the factors which prevent the attainment of the ideal. As stated in the previous chapter, the majority of the carrier servo systems employ suppressed-carrier modulated signals; therefore, all further discussion is limited to such systems.

2.1. Desired Transfer Characteristics

A suppressed-carrier modulated signal may be expressed as

$$e_s(t) = e_d(t) \times k_c \cos(\omega_c t + \phi_c)$$

where

$e_s(t)$ = instantaneous signal voltage,
$e_d(t)$ = instantaneous data or modulating voltage,
k_c, ω_c, and ϕ_c = amplitude, frequency, and phase of the carrier voltage.

For a sinusoidal modulating voltage,

$$e_d(t) = k_d \cos(\omega_d t + \phi_d),$$
$$e_s(t) = k_c k_d \cos(\omega_c t + \phi_c) \cos(\omega_d t + \phi_d).$$

By appropriate trigonometric manipulation the expression for $e_s(t)$ may be rewritten as

$$e_s(t) = \frac{k_c k_d}{2} \{\cos[(\omega_c + \omega_d)t + \phi_c + \phi_d] + \cos[(\omega_c - \omega_d)t + \phi_c - \phi_d]\},$$

which shows that for every data frequency there appear two signal frequencies which are equal to the sum and the difference of that data frequency and the carrier frequency. The phase of the higher signal frequency is equal to the sum of the phases of the carrier and the data frequencies, whereas the phase of the lower frequency is the difference of the same two quantities. The amplitudes of the two signal frequencies are equal.

Now suppose that a servo design problem has been completed and that the desired compensation function is known on a non-carrier basis, that is, the desired data-frequency transfer characteristic is known. The question arises as to how that transfer characteristic will appear on a carrier basis. The last equation in the preceding paragraph presents evidence which points towards the answer to this problem. As the amplitudes of the resulting signal frequencies in a modulated signal are equal, a plot of the amplitudes of the signal frequencies versus frequency for various data frequencies will form a curve which is symmetrical about the carrier frequency. On the other hand, a plot of the phase of the signal frequencies versus frequency will form a curve which is skew-symmetrical about the carrier frequency. With this as a basis, the logical answer to the problem of the required carrier-compensation transfer characteristic can be deduced to be that the transfer characteristic should have:

1. An amplitude response which is symmetrical about the carrier frequency and which is similar to the desired data-frequency transfer characteristic for the signal frequencies equal to the sum of the carrier frequency and the respective data frequencies.

2. A phase response which is skew-symmetrical about the carrier frequency and which is similar to the desired data-frequency transfer characteristic for the signal frequencies equal to the sum of the carrier frequency and the respective data frequencies.

As the signal frequencies are the sum and the difference of the carrier frequency and the respective data frequencies, the response curves of a carrier-compensation device, in accordance with this discussion, will be symmetrical when plotted against an arithmetical frequency scale.

A more rigorous consideration of the problem of carrier-compensation transfer characteristics is given by Newton (6). In a set of notes, which was distributed in April 1948, he derives an expression showing that the output of a carrier-compensation device for a sinusoidally modulated, suppressed-carrier input signal is

$$e_2(t) = k_c k_d[X \cos(\omega_d t + \phi_x) \cos(\omega_c t + \phi_R) - Y \sin(\omega_d t + \phi_y) \sin(\omega_c t + \phi_R)],$$

where X, Y, ϕ_x, and ϕ_y are functions of the data frequency ω_d, the phase of data frequency ϕ_d, the phase of the carrier frequency ϕ_c, an arbitrary phase angle ϕ_R, and the data-frequency transfer characteristic of the carrier network $\overset{v}{H}(j\omega)$. This equation indicates that the output will normally contain two voltages which are in phase quadrature. As the quadrature component is not desired, steps may be taken to obtain devices with characteristics such as to make this component voltage equal to, or approach, zero. A study of the equations presented by Newton indicates that one method by which this may be accomplished is to have the signal-frequency transfer characteristic of a carrier-compensation device equal to the desired data-frequency transfer characteristic and symmetrical about the carrier-frequency, the same situation as stated in the previous paragraph; however, there are other signal-frequency transfer characteristics which will also give the desired result. The first-mentioned method yields a characteristic

$$\overset{v}{H}[j(\omega_c + \omega_d)] = H(j\omega_d),$$

where

$\overset{v}{H}[j(\omega_c + \omega_d)]$ = the signal-frequency transfer-characteristic of the carrier-compensation device,

$H(j\omega_d)$ = the data-frequency transfer characteristic about zero frequency.

For

$$\omega = \omega_c + \omega_d,$$

$$\overset{v}{H}(j\omega) = H[j(\omega - \omega_c)] = H(j\omega - j\omega_c).$$

If $H(j\omega)$, that is, the data-frequency transfer characteristic, about zero frequency, is the transfer characteristic of a physically realizable structure, then $H(j\omega)$ cannot be physically realized as it is equivalent to the complex function $H(j\omega - j\omega_c)$. As the same is true for the other carrier-compensation transfer characteristics, the desired characteristics only can be approximated by physically realizable structures for applications to servo systems.

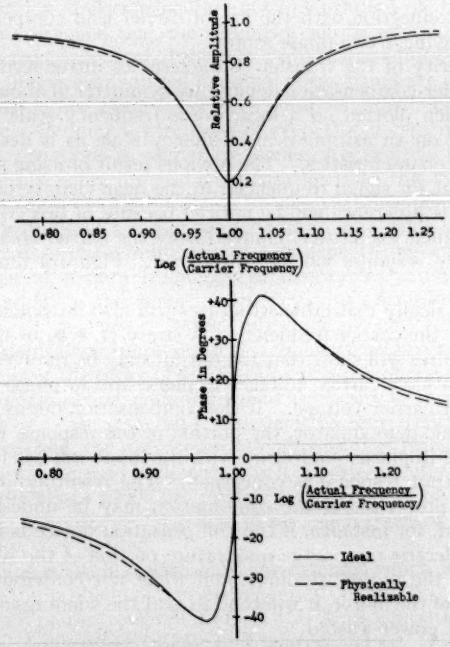

FIG. 4. Carrier lead transfer characteristics.

Figure 4 illustrates the ideal transfer characteristic for a carrier lead-compensation device,* the data-transfer properties of which are similar to those illustrated in Fig. 2. For comparison, the response of a physically realizable carrier lead network is also shown.

2.2. *Limitations Imposed by Use of Physically Realizable Characteristics*

As the ideal carrier-compensation characteristic for a particular application cannot be physically realized, the limitations imposed by the use of physically realizable characteristics must be considered.

Naturally, the limited scope of this paper prevents the consideration of all possible solutions to the compensation problem; therefore, the assumption must be made that the system performance will be impaired by the use of a characteristic other than the ideal. An interesting variation to this assumption exists in certain systems where the non-ideal properties of the compensation device match the non-ideal properties of other devices in the system, resulting in a system which is a fair approximation to the ideal. Heuchling (4) has considered this situation in connection with the use of carrier lead compensation in a system employing a two-phase motor.

The majority of the transfer characteristics introduced by the use of simple carrier-compensation devices are symmetrical about the carrier frequency when plotted on a logarithmic frequency scale rather than when plotted on an arithmetical frequency scale as is desired for the ideal transfer characteristics. The obvious result of using such characteristics is that for signal frequencies in the near vicinity of the carrier frequency little difference will be noticed because of this error, whereas for signal frequencies farther removed from the carrier frequency inaccuracies in the response will be introduced. Figure 4 illustrates this situation.

Many physically realizable structures will also introduce unwanted phase shift of the carrier frequency. A study (7, 8, 9) of the response of such structures will show that the output may be resolved into components which have carrier voltages in phase and in phase quadrature with the input carrier voltage. If the compensation device is followed by an electronic demodulator, the portion of the response which has a carrier voltage in phase quadrature with the input will be eliminated as far as the output response is concerned. The resulting effect on the data-transfer properties of the combination may be undesirable. On the other hand, for instance, if the compensation device is followed by a two-phase electric motor, the quadrature portion of the response will be present in the motor windings, and while not contributing to the useful output of the motor, it will tend to heat the windings and increase the amount of power wasted.

One of the greatest limitations imposed by physical structures is the inability to obtain accurately even the approximations to the ideal transfer characteristics. In passive electrical networks the elements must be held to very close tolerances[7] if accurate response is desired. In electromechanical devices the precision with which desired responses may be obtained is even less.

2.3. *Limitations Imposed by Carrier-Frequency Instability*

Carrier-compensation devices may be divided effectively into two classes; namely, those which operate on the carrier signal directly, and

[7] See last paragraph of Section 4.3, Part II, for an example.

those which operate on the carrier signal indirectly by demodulating the signal, introducing the compensation, and then remodulating the signal. In the latter case the compensation is essentially independent of the carrier frequency; however, in the former case the compensation is not independent of the carrier frequency, and very definite limitations are imposed on the types of compensation transfer characteristics which can be obtained. Electrical networks are the main type of the devices which operate on the carrier signals directly.

Compensation devices which are not independent of the carrier frequency are so designed that the responses will be approximately what is desired on the basis of an absolutely stable carrier frequency. Often the carrier voltages encountered in actual applications are not frequency stabilized any closer than 1 per cent, and for small, portable servo systems carrier-frequency variations of even 10 per cent must be tolerated. Such variations in carrier frequency immediately eliminate the possibility of obtaining integral compensation with this type of device. For example, suppose that a carrier-compensation network could be built which would give a response that had an amplitude which was ten times higher at a data frequency of zero (the theoretical carrier frequency) than at a data frequency of two cycles per second (signal frequencies equal to the carrier frequency plus and minus two cycles per second). This network would introduce integral compensation if the carrier maintained its theoretical frequency; however, if the carrier frequency was of the order of 400 cycles per second and it drifted 1 per cent, the compensation network would introduce a response characteristic with a peak at a data frequency of four cycles per second. Similarly reasoned, the only types of compensation transfer characteristics which may be used in devices which operate directly on the carrier signals are the types which are relatively independent of small changes in the carrier frequency. Needless to say, such types of transfer characteristics are very limited.

The author is familiar with only one attempt which has been made to minimize the problem of carrier-frequency instability. This attempt by a member of the research staff of the Massachusetts Institute of Technology is discussed briefly in Chapter 5, Part II of this paper. Often the problem of carrier-frequency instability is simply avoided by the use of compensation devices which act on the carrier signals only indirectly.

2.4. *Summary of Basic Requirements for Carrier-Compensation Devices*

The fundamental requirement for a carrier-compensation device is that it presents a close approximation to a non-physically-realizable function of a complex frequency variable as indicated in Section 2.1. In the obtaining of this approximation the effects on the response of variations in the carrier frequency and of components of the device

must be considered, as well as the range of the data frequencies which must be accurately compensated. Specifically, a carrier-compensation device must be able to produce the desired compensation over the range of data frequencies of interest regardless of expected variations in the carrier frequency and changes which will be introduced into the response by variations in the components of the device.

3. R-L-C NETWORKS

In the previous chapter, the desired types of transfer characteristics for carrier-compensation devices were described. The next question to be considered is how suitable transfer characteristics can be realized for particular applications. Two possible solutions are now available in the field of passive electrical networks: Networks involving resistances, inductances, and capacitances (R-L-C networks), and networks involving only resistances and capacitances (R-C networks). As the former

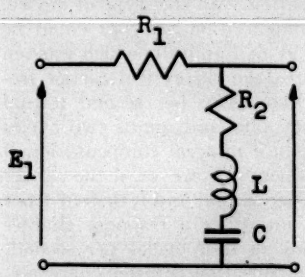

FIG. 5. Carrier compensation network.

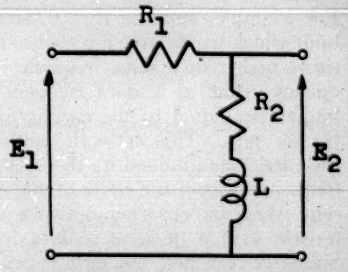

FIG. 6. Non-carrier compensation network.

class of networks is the more general, it will be treated in the present chapter, and R-C networks will be considered in a later chapter.

3.1. Transfer Characteristics Obtainable

As R-L-C networks are the most general class of networks, they present the possibility of theoretically obtaining any transfer characteristic which can be physically realized with linear, passive elements. Actually, physical restrictions on the elements, such as the non-existence of a pure inductance, limit the accuracy with which the desired transfer characteristics can be obtained, but in general the effects of these physical restrictions may be minimized by variations in the design of the networks.

The variety of transfer characteristics utilized in servo compensation include many which make use of the resonant properties of a parallel or series L-C circuit. Perhaps the simplest example is the network which consists of a shunt R-L-C branch (Fig. 5). In this circuit, which

is essentially a band-elimination filter, the series L-C circuit is made resonant at the carrier frequency, thereby causing the magnitude of E_2/E_1 to be a minimum at that frequency. The frequency-response characteristic of this circuit about the carrier frequency is a good approximation to the characteristic which is required for a carrier-system lead network.

By noting the similarity between the carrier-system lead network and the non-carrier lead network (Fig. 6), one advantage of R-L-C networks may be deduced. The impedance of the shunt branch of the non-carrier network is a minimum at zero frequency, the "carrier frequency" in this case, and the impedance of the shunt branch of the carrier network is a minimum at the designated non-zero carrier frequency. In both cases for frequencies higher than the carrier frequencies, the shunt branch presents a predominantly inductive reactance in series with the resistance. The difference between the two networks is that the carrier network has had the inductance of the non-carrier network replaced by a series L-C circuit which is resonant at the desired carrier frequency. This change effectively translates the properties of the simple R-L circuit, insofar as its effect on data frequencies is concerned, from the zero carrier frequency basis to the non-zero carrier frequency basis.

The idea mentioned in the previous paragraph for translating a network from the zero carrier-frequency basis to a non-zero carrier-frequency basis is used frequently in obtaining the desired transfer characteristic in a carrier network. The network is first designed on the zero carrier-frequency basis, which with the available knowledge of network-synthesis procedures is normally not a difficult task. Then the following simple rules (2, p. 190) are applied to translate the network to the desired carrier frequency:

1. Leave resistances unaltered.
2. Replace each condenser with a parallel L-C circuit, the condenser of which is one-half the value of the original condenser, and make the circuit resonant at the desired carrier frequency.
3. Replace each inductance with a series L-C circuit, the inductance of which is one-half the value of the original inductance, and make the circuit resonant at the desired carrier frequency.

3.2. Types of Networks Employed

In the synthesis of the R-L-C networks several configurations of network elements may result, for example, the lattice, the ladder, the simpler "T" or "π," or some combination of these structures. The first-mentioned example, the lattice, often cannot be used in servo systems as it is applicable only for use in circuits which are balanced to ground; however, except in some special cases, transfer characteristics which are

useful in servo compensation may be realized in some alternate form besides the lattice in order to overcome this difficulty.

3.3. Non-linearities Introduced by Inductances

As mentioned in Section 3.1, one limitation in the realization of the desired transfer characteristic in an R-L-C network is the nonexistence of a pure inductance. Actual inductances have d-c. coil resistance, hysteresis and eddy-current losses, and distributed winding capacitances all of which are often neglected in network design, but these factors may have great influence on the actual response of the network when it is constructed and tested. To further complicate the problem, the hysteresis and eddy-current losses, and sometimes the value of the actual inductance are non-linear functions of the input; hence, circuits incorporating inductances do not present a sound basis for the linear analysis method of network design.

The d-c. coil resistance associated with an inductance is normally small compared to the inductive reactance of a well designed inductance and will not cause a very great change in the network response. In many cases, this d-c. coil resistance may be included as a part of the resistance required in the circuit, and thereby eliminated as a factor contributing to the error in the network response.

The distributed capacitance in an inductance is usually effective only at frequencies much higher than those of interest in a carrier network; hence it may be justifiably neglected in many cases.

Hysteresis and eddy-current losses in an inductance do present important problems in accurate network synthesis. Heuchling (4) has considered this problem with particular reference to the design of R-L-C networks for use in conjunction with a two-phase-motor servo system, and he has shown how an equivalent circuit may be used to account for the eddy-current loss in the inductances and how an approximation may be made to include the effect of the hysteresis loss.

The artifices developed by Heuchling, together with the simpler methods of accounting for the coil resistance and distributed capacitance, may be used when the accurate design of an R-L-C network is necessary; however, with many of the networks which the servo designer will encounter the easier and more direct method is to design the networks on basis of the availability of pure elements and then adjust the elements as necessary on the basis of experimentally obtained network response characteristics. This latter procedure consists essentially of first varying the proper inductances or capacitances to obtain resonances at the required frequencies in the series or parallel L-C circuits, and finally varying the remainder of the components slightly, if necessary, to obtain the desired response at a selected number of frequency points. A complete frequency response of the network will verify the adjustment of the components.

3.4. *Advantages of R-L-C Networks*

The inherent advantage of the R-L-C networks is the availability of the wide variety of transfer characteristics due to the possibility of synthesizing any desired transfer characteristic. Experience has also shown that in most cases where networks can be synthesized as either R-L-C or R-C networks fewer elements are required for the former. This is a distinct advantage in cases where space requirements are important, although the weight of the inductances may at times reduce the advantages of a smaller number of elements. The smaller number of elements is also an advantage when an attempt is made to adjust the network on the basis of laboratory tests.

3.5. *Disadvantages of R-L-C Networks*

The main disadvantage of the R-L-C networks is the non-linearities introduced by actual inductances. Although methods are available to compensate for these non-linearities during the design of the network, the methods are by no means simple or accurate and are to be avoided when possible. Secondary disadvantages exist in certain cases because of the weight of the inductances and the presence of the magnetic fields which they create.

(*To be continued.*)

Colored Aluminum Chain.—A new group of styled chains, made of aluminum and cleverly finished by means of their Mirrodip JF Electrodized processes to simulate Gold, Silver, Nickel, Brass, Copper, Chromium, red, blue, gray, green, brown, black and the various pastel shades, have recently been developed by Colonial Alloys Co., Philadelphia, Pa.

The finished chain surfaces have a much higher corrosion, abrasion and hardness resistance, and in comparative tests have been found to outwear any of the electro-plated or lacquered finishes.

So close is the simulation to the color of any karat of gold for example, that it is difficult even for an expert to recognize it at first sight. This non-precious metal is tax free. There is also an economic advantage over the usual chain made of brass and electroplated, enameled or lacquered. Colonial chain is about two thirds lighter in weight than the same styles in brass or steel.

From the smallest diameter of bead type to the largest Elwel twist type of wire chain it is processed in long lengths and packaged in hanks and on spools.

Colonial chain should find wide acceptance for costume jewelry in necklaces, wrist bands, bracelets, belts, novelties, handbags, keys, identification pieces, instruments, dress trimmings, leather goods, lighting fixtures and chandeliers, pocket watches, fobs, small dogs, shower curtain and portier chains, venetian blinds, bathtub stoppers, canteen, bird cage chains, in millinery and in the dress trades; and many other applications where chain can serve, decoratively and functionally.

Nickel-Clad Copper Conductor.—For many years electrical designers and maintenance men have been looking for a really tough conductor—one able to withstand abusive conditions, yet having good electrical characteristics as well. Such a conductor is now available through the Alloy Metal Wire Co.

The outstanding advantages of Nickel-clad copper conductor are strength, corrosion-resistance, and resistance to high-temperature failure. Its electrical conductivity is 70 per cent that of copper alone.

Among the many applications where Nickel-clad copper conductor is being used with outstanding success are: lead wires for electronic tubes, heating applicances, infra-red ovens, resistance and induction furnaces. It is also used for special power lines, fixture wiring in corrosive atmospheres and spark plug electrodes.

For electronic tube uses in particular, Nickel-clad copper wire offers many important advantages. It provides excellent conductivity for uses where exposed copper is not desirable. It does not oxidize, flake, or embrittle under the high temperatures encountered during stem making, sealing and exhausting. Welds are strong and flexible. Nickel-clad copper wire can be welded to molybdenum and tungsten as well as to itself.

Nickel-clad copper conductor is available in rods from $\frac{1}{2}$ to 0.031 in. in diameter; and in round and flat wire, coils or spools, solid or stranded, in all commonly used sizes.

INTERFLECTION TABLES FOR ROOMS WITH FLOORS OF HIGH REFLECTANCE

BY

JAMES S. Y. LEE[1]

1. INTRODUCTION

Previous interflection tables[2] have given the necessary data for the design of lighting installations. These tables include the effects of multiple reflections among walls, ceiling, and floor, these interflections having been taken into account by the solution of an integral equation. Values were given for the interflectance f and the ratios H_{1T}/D_{AV}, H_{1M}/D_{AV}, H_2/D_{AV}, and H_3/D_{AV} for floor reflectances of 0.10 and 0.30.

Recent investigations show that floor reflectances even higher than 0.30 are advantageous. Thus there seems to be a growing need for interflection tables for higher floor reflectances. In such cases the usual procedure has been to extrapolate from the available tables, but this process is questionable since the previous values for floor reflectance (0.10 and 0.30) do not provide an adequate basis for extrapolation. The purpose of this paper, therefore, is to compute new tables for floor reflectance of 0.50, which will allow interpolation for all ordinary rooms. The effect of high floor reflectance on the light distribution in rooms may also be ascertained.

2. TABLES

Since ceiling lighting has been shown[3] to be definitely superior visually to other types of lighting, this paper will be confined to Types II*a* and II*b* only. Type II*a* is lighting from a complete luminous ceiling, while II*b* applies to reflecting ceilings that act as secondary sources. The equations were obtained from a previous paper[2] but were slightly simplified. They are as follows:

$$f = D\cosh\left(\tfrac{3}{2}k_r\sqrt{1-\rho_1}\right) - E\left[(2-\rho_1(1+\rho_3))\cosh\left(2k_r\sqrt{1-\rho_1}\right) + 2\sqrt{1-\rho_1}\sinh\left(2k_r\sqrt{1-\rho_1}\right)\right]\sinh\left(\tfrac{3}{2}k_r\sqrt{1-\rho_1}\right), \quad (1)$$

$$\frac{H_{1T}}{D_{AV}} = \frac{B_2}{f}, \quad (2)$$

[1] Graduate Student, Department of Physics, Brown University, Providence, R. I.

[2] PARRY MOON AND D. E. SPENCER, "Light Distributions in Rooms," JOUR. FRANKLIN INST., Vol. 242, p. 111 (1946); "Lighting Design by the Interflection Method," Ibid., Vol. 242, p. 465 (1946); "Lighting Design," Cambridge, Mass., Addison-Wesley Press (1948).

[3] PARRY MOON AND D. E. SPENCER, "An Engineering Correlation of Room Colors," JOUR. FRANKLIN INST., Vol. 247, p. 117 (1949).

$$\frac{H_{1M}}{D_{AV}} = \frac{B_2 \cosh (k_r\sqrt{1-\rho_1}) + C_2 \sinh (k_r\sqrt{1-\rho_1})}{f}, \tag{3}$$

$$\frac{H_2}{D_{AV}} = \frac{D}{f}, \tag{4}$$

$$\frac{H_3}{D_{AV}} = \frac{2\rho_3\sqrt{1-\rho_1}E}{f}, \tag{5}$$

where

$$\begin{aligned}
D &= E[(2 - \rho_1(1+\rho_3)) \sinh (2k_r\sqrt{1-\rho_1}) \\
&\qquad + 2\sqrt{1-\rho_1} \cosh (2k_r\sqrt{1-\rho_1})], \\
E &= \{[2(1+\rho_2\rho_3) - \rho_1(1+\rho_2)(1+\rho_3)] \sinh (2k_r\sqrt{1-\rho_1}) \\
&\qquad + 2\sqrt{1-\rho_1}(1-\rho_2\rho_3) \cosh (2k_r\sqrt{1-\rho_1})\}^{-1}, \\
B_2 &= \rho_1 E[(1-\rho_3) \sinh (2k_r\sqrt{1-\rho_1}) \\
&\qquad + \sqrt{1-\rho_1}(1+\rho_3) \cosh (2k_r\sqrt{1-\rho_1})], \\
C_2 &= -\rho_1 E[(1-\rho_3) \cosh (2k_r\sqrt{1-\rho_1}) \\
&\qquad + \sqrt{1-\rho_1}(1+\rho_3) \sinh (2k_r\sqrt{1-\rho_1})].
\end{aligned} \tag{6}$$

In these equations, subscripts 1, 2, and 3 refer to walls, ceiling, and floor, respectively, and

f = interflectance (Type IIa lighting)
$= \dfrac{\text{pharos on horizontal surface at table level}}{\text{pharos from luminaire}}$;

H_{1T} = helios at top of walls, including interflections;

H_{1M} = helios at middle of walls, including interflections;

H_2 = helios of ceiling, including interflections;

H_3 = helios of floor, including interflections;

D_{AV} = average pharosage incident on top of an imaginary plane surface at table level (taken as ¼ ceiling height in this paper);

k_r = domance $= \dfrac{\text{total wall area}}{4\ (\text{floor area})}$;

ρ_1 = average wall reflectance;

ρ_2 = average ceiling reflectance;

ρ_3 = average floor reflectance.

Computations were made by means of an electrically driven computing machine to five figures, and results were checked by plotting. The final tables are rounded off to four figures. The calculated data are listed in Tables I to VI.

TABLE I.—*Ceiling Lighting, Type IIa, f.*

	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	1.6667	1.6667	1.6667	1.6667	1.5384	1.5384	1.5384	1.3333	1.3333	1.3333
0.1	1.5308	1.3730	1.2861	1.2104	1.2906	1.2180	1.1539	1.1522	1.1014	1.0554
0.2	1.4102	1.1601	1.0401	9443	1.1036	1.0000	9157	1.0057	9286	8635
0.3	1.3018	9968	8652	7660	9561	8398	7503	8834	7934	7207
0.4	1.2041	8666	7328	6361	8361	7158	6269	7813	6840	6093
0.5	1.1156	7597	6281	5359	7361	6161	5302	6930	5934	5193
0.7	9613	5938	4717	3901	5785	4649	3882	5500	4520	3829
1.0	7739	4198	3156	2495	4105	3121	2485	3930	3053	2467
2.0	3860	1410	^{1}8816	^{1}5948	1382	^{1}8730	^{1}5933	1330	^{1}8571	^{1}5890

NOTE: Superscripts refer to number of zeros after decimal point. When no superscript is used, a zero is understood. For example, 1906 = 0.1906, ^{1}6335 = 0.06335.

TABLE II.—*Ceiling Lighting, Type IIb, f.*

	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	1.3333	1.3333	1.3333	1.3333	1.0769	1.0769	1.0769	6666	6666	6666
0.1	1.2246	1.0984	1.0289	9683	9034	8526	8077	5761	5507	5277
0.2	1.1282	9281	8321	7554	7725	7000	6410	5029	4643	4317
0.3	1.0414	7974	6922	6128	6693	5879	5252	4417	3967	3604
0.4	9633	6933	5863	5089	5853	5010	4388	3906	3420	3046
0.5	8925	6077	5025	4287	5153	4313	3712	3465	2967	2597
0.7	7690	4751	3773	3121	4049	3255	2718	2750	2260	1914
1.0	6191	3358	2525	1996	2873	2185	1740	1965	1526	1234
2.0	3088	1128	^{1}7053	^{1}4758	^{1}9676	^{1}6111	^{1}4153	^{1}6650	^{1}4286	^{1}2945

TABLE III.—*Ceiling Lighting, Type II, H_{1T}/D_{AV}.*

	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	6000	3750	2250	^{1}7500	3750	2250	^{1}7500	3748	2250	^{1}7500
0.1	6297	3914	2340	^{1}7769	3914	2340	^{1}7769	3914	2340	^{1}7769
0.2	6629	4139	2482	^{1}8267	4139	2482	^{1}8267	4139	2482	^{1}8267
0.3	6996	4427	2676	^{1}8988	4427	2676	^{1}8988	4427	2676	^{1}8988
0.4	7400	4775	2923	^{1}9935	4775	2923	^{1}9935	4775	2923	^{1}9935
0.5	7843	5190	3225	1112	5190	3225	1112	5190	3225	1112
0.7	8849	6226	4010	1427	6226	4010	1427	6226	4010	1427
1.0	1.0694	8390	5722	2142	8390	5722	2142	8390	5722	2142
2.0	2.0758	2.4130	1.9968	8815	2.4130	1.9968	8815	2.4130	1.9968	8815

TABLE IV.—*Ceiling Lighting, Type II, H_{1M}/D_{AV}.*

	ρ_3=0.80 (Ceiling)				ρ_3=0.70			ρ_3=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	6000	3750	2250	[1]7500	3750	2250	[1]7500	3750	2250	[1]7500
0.1	6083	3765	2244	[1]7431	3765	2244	[1]7431	3765	2244	[1]7431
0.2	6172	3792	2250	[1]7417	3792	2250	[1]7417	3792	2250	[1]7417
0.3	6266	3832	2267	[1]7453	3832	2267	[1]7453	3832	2267	[1]7453
0.4	6366	3883	2293	[1]7533	3883	2293	[1]7533	3883	2293	[1]7533
0.5	6471	3945	2330	[1]7654	3945	2330	[1]7654	3945	2330	[1]7654
0.7	6696	4099	2430	[1]8009	4099	2430	[1]8009	4099	2430	[1]8009
1.0	7074	4405	2639	[1]8795	4405	2639	[1]8795	4405	2639	[1]8795
2.0	8669	5986	3801	1337	5986	3801	1337	5986	3801	1337

TABLE V.—*Ceiling Lighting, Type II, H_2/D_{AV}.*

	ρ_3=0.80 (Ceiling)				ρ_3=0.70			ρ_3=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0626	1.1004	1.1252	1.1497	1.1004	1.1252	1.1497	1.1004	1.1252	1.1497
0.2	1.1306	1.2148	1.2694	1.3233	1.2148	1.2694	1.3233	1.2148	1.2694	1.3233
0.3	1.2041	1.3441	1.4349	1.5243	1.3441	1.4349	1.5243	1.3441	1.4349	1.5243
0.4	1.2836	1.4899	1.6242	1.7566	1.4899	1.6242	1.7566	1.4899	1.6242	1.7566
0.5	1.3693	1.6539	1.8400	2.0250	1.6539	1.8400	2.0250	1.6539	1.8400	2.0250
0.7	1.5612	2.0423	2.3658	2.6926	2.0423	2.3658	2.6926	2.0423	2.3658	2.6926
1.0	1.9061	2.8149	3.4554	4.1305	2.8149	3.4554	4.1305	2.8149	3.4554	4.1305
2.0	3.7460	8.2292	12.2187	17.1728	8.2292	12.2187	17.1728	8.2292	12.2187	17.1728

TABLE VI.—*Ceiling Lighting, Type II, H_3/D_{AV}.*

	ρ_3=0.80 (Ceiling)				ρ_3=0.70			ρ_3=0.50		
k_r	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
	ρ_3=0.50 (Floor)									
0	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
0.1	4901	4846	4810	4774	4846	4810	4774	4846	4810	4774
0.2	4803	4695	4625	4557	4695	4625	4557	4695	4625	4557
0.3	4707	4548	4447	4350	4548	4447	4350	4548	4447	4350
0.4	4612	4405	4274	4152	4405	4274	4152	4405	4274	4152
0.5	4519	4265	4108	3963	4265	4108	3963	4265	4108	3963
0.7	4337	3994	3793	3609	3994	3793	3609	3994	3793	3609
1.0	4076	3620	3361	3135	3620	3361	3135	3620	3361	3135
2.0	3300	2581	2233	1956	2581	2233	1956	2581	2233	1956

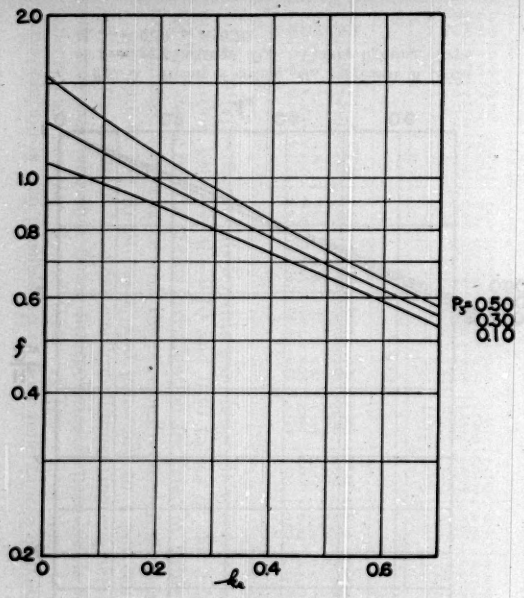

FIG. 1. Interflectance as a function of domance k_r:

$$k_r = \frac{\text{total wall area}}{4\ (\text{floor area})}.$$

Curves are for wall reflectance $\rho_1 = 0.50$, ceiling reflectance $\rho_2 = 0.70$, and for ceiling lighting Type II*a*. The three curves are for the floor reflectances, $\rho_3 = 0.10, 0.30, 0.50$.

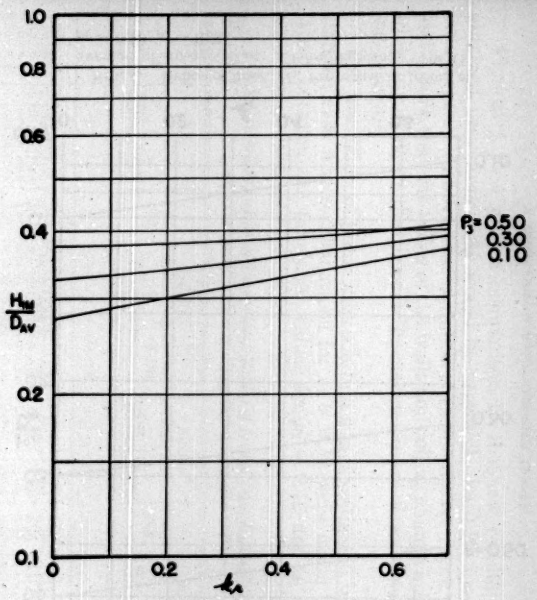

FIG. 2. Helios of walls midway between ceiling and floor, H_{1M}, expressed in terms of average pharosage, D_{AV}. Ceiling lighting, Type II, $\rho_1 = 0.50$, $\rho_2 = 0.70$.

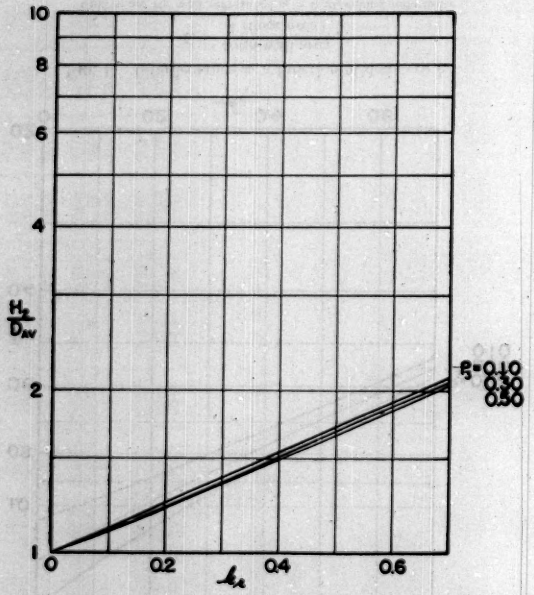

FIG. 3. Helios of ceiling, H_2, expressed in terms of average pharosage, D_{AV}. Ceiling lighting, Type II, $\rho_1 = 0.50$, $\rho_2 = 0.70$.

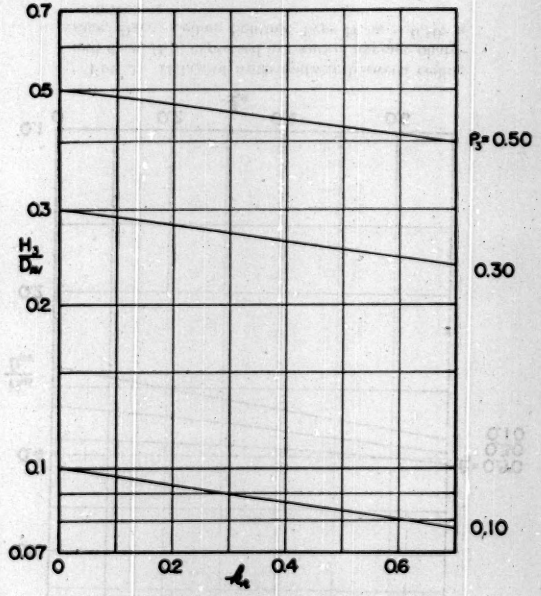

FIG. 4. Helios of floor, H_3, expressed in terms of average pharosage, D_{AV}. Ceiling lighting, Type II, $\rho_1 = 0.50$, $\rho_2 = 0.70$.

3. COMPARISON WITH PREVIOUS TABLES

Figures 1 to 4 give a comparison of the new results with those obtained previously. As would be expected, the 0.50 floor raises the interflectance f above that obtained with the darker floors. Figures 2 and 3 show that the helios of the walls and ceiling change inappreciably, while Fig. 4 shows that floor helios varies almost directly with the floor reflectance.

The possibility of linear interpolation was investigated by taking an extreme case where calculations for $\rho_3 = 0.30$ were made by interpolating between the tabulated values for $\rho_3 = 0.10$ and $\rho_3 = 0.50$. If

TABLE VII.—*Comparison of Interpolated Values of* f, H_{1M}/D_{AV}, H_2/D_{AV}, *and* H_3/D_{AV} *with Computed Values at Floor Reflectance* $\rho_3 = 0.30$.

ρ_1	ρ_2	f		% Diff.	H_{1M}/D_{AV}		% Diff.	H_2/D_{AV}		% Diff.	H_3/D_{AV}		% Diff.
		Interpolated	Tables		Interpolated	Tables		Interpolated	Tables		Interpolated	Tables	
					$k_r = 0$								
0.80	0.80	1.3768	1.3158	4.43	0.5200	0.5200	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
0.10	0.80	1.3768	1.3158	4.43	0.0650	0.0650	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
0.50	0.70	1.3067	1.2658	3.13	0.3250	0.3250	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
0.10	0.70	1.3067	1.2658	3.13	0.0650	0.0650	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
0.50	0.50	1.1929	1.1765	1.37	0.3250	0.3250	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
0.10	0.50	1.1929	1.1765	1.37	0.0650	0.0650	0.00	1.0000	1.0000	0.00	0.3000	0.3000	0.00
					$k_r = 0.5$								
0.80	0.80	0.9951	0.9783	1.69	0.5994	0.5984	0.16	1.4175	1.4183	0.06	0.2696	0.2663	1.22
0.10	0.80	0.5212	0.5207	0.10	0.07169	0.07166	0.04	2.0300	2.0300	0.00	0.2376	0.2373	0.13
0.50	0.70	0.6959	0.6932	0.39	0.3673	0.3670	0.08	1.6818	1.6820	0.01	0.2550	0.2531	0.75
0.10	0.70	0.5174	0.5171	0.06	0.07169	0.07166	0.04	2.0300	2.0300	0.00	0.2376	0.2373	0.13
0.50	0.50	0.6634	0.6618	0.24	0.3673	0.3670	0.08	1.6818	1.6820	0.01	0.2550	0.2531	0.75
0.10	0.50	0.5100	0.5106	0.01	0.07169	0.07166	0.04	2.0300	2.0300	0.00	0.2376	0.2373	0.13
					$k_r = 1.0$								
0.80	0.80	0.7189	0.7129	0.83	0.6814	0.6806	0.12	1.987	1.989	0.14	0.2420	0.2368	2.15
0.10	0.80	0.2481	0.2481	0.00	0.08562	0.08561	0.01	4.138	4.138	0.00	0.1879	0.1875	0.21
0.50	0.70	0.4002	0.3997	0.12	0.4265	0.4261	0.09	2.861	2.861	0.00	0.2159	0.2132	1.25
0.10	0.70	0.2473	0.2473	0.00	0.08562	0.08561	0.01	4.138	4.138	0.00	0.1879	0.1875	0.21
0.50	0.50	0.3843	0.3840	0.08	0.4265	0.4261	0.09	2.861	2.861	0.00	0.2159	0.2132	1.25
0.10	0.50	0.2457	0.2472	0.61	0.08562	0.08561	0.01	4.138	4.138	0.00	0.1879	0.1875	1.21

linear interpolation holds for this wide range, it will certainly hold for interpolation between the 0.10 and 0.30 tables or the 0.30 and 0.50 tables. Some results of this investigation are listed in Table VII. The interpolated values agree with the previously tabulated values to within 5 per cent in the worst cases, and generally to 1 per cent. This is sufficiently accurate for most work in illuminating engineering. Thus one may conclude that linear interpolation is permissible for all floor reflectances between 0 and 0.50. A few calculations indicate that extrapolation is also possible up to floor reflectances of 0.80.

4. APPROXIMATE FORMULAS

Simplified formulas are available for the approximate calculation of interflections.[4] Since these formulas were based mainly on the lower floor reflectances, it seemed advisable to see how close an approximation they would give for $\rho_3 = 0.50$. The approximate equations are,

$$f = \frac{1}{1 - \rho_2\rho_3} e^{-ak_r} \tag{7}$$

$$\frac{H_{1M}}{D_{AV}} = \frac{\rho_1}{2}(1 + \rho_3)e^{\beta_1 k_r} \tag{8}$$

$$\frac{H_2}{D_{AV}} = e^{\beta_2 k_r} \tag{9}$$

$$\frac{H_3}{D_{AV}} = \rho_3 e^{-\beta_3 k_r}, \tag{10}$$

where

$$\begin{aligned} \alpha &= 1.530 + 1.20\rho_3 - 0.972\rho_1 - 0.820(\rho_1)^2 \\ \beta_1 &= 0.418 + 0.116\rho_1 - 0.772\rho_3 \\ \beta_2 &= 1.608 - 0.160\rho_3 - 1.040\rho_1 \\ \beta_3 &= 0.544 - 0.130\rho_3 - 0.330\rho_1. \end{aligned} \tag{11}$$

A comparison between values computed by these approximate formulas and the tabulated values is given in Table VIII. The discrepancies are largest for f, and may be 10 per cent or more. The results, however, are very good for the recommended wall reflectance of 0.50 and ceiling reflectance of 0.70. The other quantities are approximated much more closely, in all cases with an error of less than 5 per cent.

TABLE VIII.—*Comparison of Values Obtained by Approximate Formulas with Computed Values at Floor Reflectance $\rho_3 = 0.50$.*

ρ_1	k_r	f			H_{1M}/D_{AV}			H_2/D_{AV}			H_3/D_{AV}		
		Approx.	Tables	% Err.	Approx.	Tables	% Err.	Approx.	Tables	% Err.	Approx.	Tables	% Err.
		$\rho_3 = 0.70$			ρ_3 = All Values			ρ_3 = All Values			ρ_3 = All Values		
0.50	0	1.5385	1.5385	0.00	0.3750	0.3750	0.00	1.0000	1.0000	0.00	0.5000	0.5000	0.00
0.10		1.5385	1.5385	0.00	0.0750	0.0750	0.00	1.0000	1.0000	0.00	0.5000	0.5000	0.00
0.50	0.3	0.9304	0.9561	2.68	0.3853	0.3832	0.51	1.353	1.344	0.65	0.4551	0.4548	0.06
0.10		0.8169	0.7503	8.16	0.07598	0.07453	1.91	1.576	1.524	3.31	0.4374	0.4350	0.54
0.50	0.7	0.4763	0.5785	17.65	0.3994	0.4099	2.56	2.025	2.042	0.84	0.4013	0.3994	0.48
0.10		0.3508	0.3882	9.65	0.07733	0.08009	3.44	2.893	2.693	6.95	0.3659	0.3609	1.38
0.50	1.0	0.2882	0.4105	29.80	0.4103	0.4405	6.85	2.740	2.815	2.66	0.3653	0.3620	0.91
0.10		0.1862	0.2485	25.10	0.7836	0.8795	10.90	4.563	4.131	10.45	0.3201	0.3155	2.06
0.50	2.0	0.5397	0.13823	61.10	0.4490	0.5984	24.90	7.508	8.230	8.78	0.2668	0.2581	3.28
0.10		0.02263	0.05933	62.00	0.08182	0.1337	39.10	20.429	17.173	15.95	0.2049	0.1956	4.55

[4] PARRY MOON AND D. E. SPENCER, "Simplified Interflection Calculations," to be published in this JOURNAL, February, 1951.

It may be said, therefore, that the approximate formulas are probably as satisfactory for 0.50 floors as for darker floors.

5. CONCLUSIONS

New interflection tables have been computed, for a floor reflectance of 0.50, to supplement the previous tables for 0.10 and 0.30. This extends the range and utility of the interflection method of lighting calculation. The paper shows also that linear interpolation can be used and that the approximate exponential formulas are satisfactory over a fairly wide range of variables, provided one does not require too high an accuracy.

The light floors considered in this paper are definitely advantageous because they increase the amount of available light, tend to decrease shadows, and reduce helios ratios.

Noncombustible Plastic Insulation (*Electrical Engineeirng*, Vol. 69, No. 9).—E. I. du Pont de Nemours and Company has announced a new plastic electrical insulation which will not support combustion. Designated "Rulan" flame-retardant plastic, the new material will not burn after the flame has been removed, and neither will it drip when molten. Its power factor, over a wide range of frequencies, is 0.002, its dielectric constant is 2.7. It can be extruded on to wire at high speeds, and can also be injection molded.

Plastic-Metal Screw (*Tele-Vision Engineering*, Vol. 1, No. 8).—Fasteners featuring a serrated metal core which has been extrusion-coated with a thermoplastic material, have been developed. The type of core and plastic used depends entirely on the use to which the screw will be put. After the extrusion process the resulting composite rod is cut in an automatic screw machine to form accurate threads and a strong head. The metal core runs the entire length of the screw and furnishes most of the screw strength. The plastic exterior is said to give the screw all of its extra insulating and sealing qualities. The metal core carries the torque applied by a screw driver or other driving instrument.

Standard sizes available from stock range in diameter from No. 8 to $\frac{1}{2}$ in. with cellulose acetate insulation; from No. 10 to $\frac{3}{8}$ in. with polyethylene; from No. 8 to $\frac{3}{8}$ in. with cellulose acetate butyrate; and, from No. 8 to $\frac{1}{2}$ in. with ethyl cellulose insulation. (Forman Insulating Screw Corp.)

Office Layout (*Mechanical Engineering*, Vol. 72, No. 9).—Length and width are the dimensions traditionally considered in office layout. Confronted with the need for increased facilities, management normally rents more space or erects its own buildings. But office space also has the third dimension of height. For Forbes Publishing Company, utilization of this third dimension has solved manifold problems while producing greater efficiency.

Forbes' solution revolves around the newly introduced Korda-Room. A development of the functional office furniture which E. I. du Pont de Nemours & Company sponsored for its own use, the Korda-Room, manufactured under license by Korda Industries, New York, N. Y., is a movable semiprivate office comprising an L-shaped desk complete with integral partitions, cabinets, desk drawers, bookcases, shelving, and other accessories. The Forbes project is said to be the world's first commercial application of the du Pont-licensed Korda-Room.

Exhaustive investigations made by du Pont, including time and motion studies of executive and subordinate personnel at work, revealed that the du Pont-pioneered functional furniture, while providing the equivalent of a desk and a reference table, reduced office-space requirements 30 per cent or more and permitted up to 35 per cent more people to occupy a given floor space.

Further, the L-shaped desk, with its built-in accessories, puts everything within arm's reach. For workers who constantly refer to records, manuals, reference data, books and similar material the new type of office furniture seemed to suggest a profitable convenience. Partitions are moved simply by moving the desks. Executives and other employees who formerly required relatively expensive private or semiprivate offices are now housed, more comfortably, in considerably reduced floor space. Over-all attractiveness of office layout has been enhanced, office noise has been decreased, and personnel efficiency and morale increased.

THE DISTRIBUTION OF PARTICLE SIZES[1]

BY

F. KOTTLER[2]

Part II[3]

THE PROBABILITY GRAPHS

We come now to the question of how to evaluate data for particle sizes by a Log-Normal Distribution. This is usually done by a purely graphical method of analysis by means of the Probability Charts,[4] which have come to be widely accepted in the past twenty odd years and which we shall also use in what follows for purposes of illustration. Their use for the analysis of data without the help of other mathematical devices is, however, open to serious objections.

There are two kinds of Probability Charts, the arithmetic Probability Chart, and the logarithmic Probability Chart. The former was designed by Hazen (**27**)[5] in 1914, and the latter was designed by Whipple (**28**) in 1916. Both have a so-called probability scale along one side which will be explained later; along the other side, the former chart has a uniform or arithmetic scale and it serves to illustrate a Normal Probability Distribution. The latter chart has a logarithmic scale along its second side. It is used for a Log-Normal Distribution.

The probability scale has not always been well understood by the users of such charts. It will therefore be explained in what follows. In Fig. 4 is reproduced a special Logarithmic Probability Chart prepared for the purpose of the present paper by the Codex Book Company, Inc., Norwood, Mass. [Appendix II.] The horizontal midline of the chart is denoted by 50 per cent, and upward or downward probability levels, symmetric with respect to 50 per cent, are marked at increasing intervals and with an increasing number of subdivisions. The scale is therefore a non-uniform one, a fact which must be well understood when using this chart.

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[1] Communication No. 1269 from the Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[2] Kodak Park Works, Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[3] Part I appeared in this JOURNAL, Vol. 250, No. 4, October, 1950, p. 339.

[4] Loveland and Trivelli (**23**) used another kind of chart besides Probability Charts. These are also open to the same objection as the Probability Charts.

[5] The boldface numbers in parentheses refer to the references appended to Part II of this paper.

This non-uniformity comes about as follows: The distance of each probability level from the midline is equal to the normal deviate, t, in Gauss's integral,

$$P = \int_{-\infty}^{t} z dt, \tag{5}$$

where z is given by Eq. 4 and t is positive for levels of P larger than 50 per cent and negative if P is smaller than 50 per cent. The value $t = 0$ corresponds to the midline 50 per cent. Obviously, t increases more rapidly than P, for example, if P increases from 50 per cent to 100 per cent, t goes from 0 to $+\infty$. Therefore, a probability interval of 1 per cent is rendered quite small if it occurs near the midline and is rendered progressively larger if it occurs at the high levels 90 per cent, 95 per cent, . . . , or the low levels 10 per cent, 5 per cent, ...

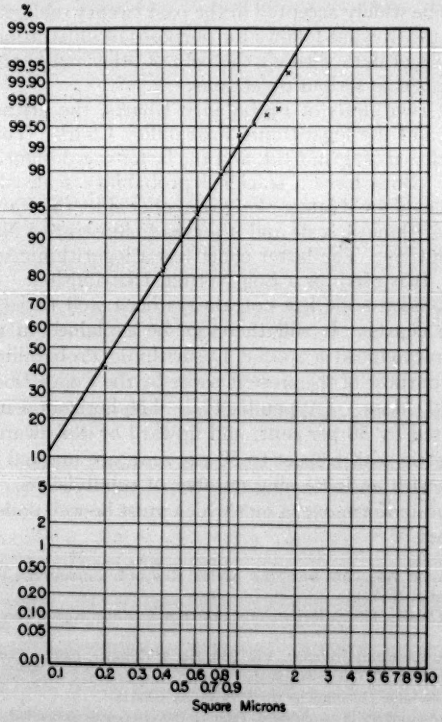

FIG. 4. Fitting a log-normal distribution graphically. (Seed Lantern No. 6102.)

These charts have one advantage. A Normal Ogive plotted on an arithmetic Probability Chart is rendered as a straight line and a Log-Normal Ogive plotted on a Log-Probability Chart is likewise represented by a straight line. This is easily understood. Consider, for instance, our problem and the logarithmic transformation, Eq. 3. This represents a linear relationship between the normal deviate, t, and the logarithm of the size, x. Consequently, it is represented graphically by a straight line if t is plotted *versus* $\ln x$. However, this is just what one achieves by the probability scale. It is really a t-scale and is merely inscribed with the P-values corresponding to the t-values.

Incidentally, this representation of the ogive by a straight line gives a clear insight into the meaning of the two parameters, a, b. The former is the intercept of that straight line with the vertical midline ($\ln x = 0$), and the latter is the slope of the straight line *versus* the horizontal line. It will be recalled that $1/b$ equals the velocity constant, k, of the growth. According to Appendix I, it is a measure of *dispersion*. It can be expressed by Karl Pearson's coefficient of variation. The value b is, therefore, a measure of *uniformity* of the distribution. The steeper the straight line the more uniform is the distribution it represents and the less is the "scatter" of the particles around the median.

The advantage described above is, however, accompanied by a serious disadvantage. This becomes manifest if experimental percentages are entered in such a chart. (See the crosses in Fig. 4.) As a rule, they will deviate somewhat from an ideal Log-Normal Distribution and will therefore not fit a straight line exactly. The question arises how to judge the fit by means of the deviations.

Any deviation in percentage occurring at the level of (say) 99 per cent or 99.5 per cent will appear much exaggerated on the chart, as compared with a deviation of the same absolute magnitude in percentage which occurs at the level 50 per cent. To be precise, the exaggeration will be 15-fold at the level of 99 per cent, 28-fold at the level of 99.5 per cent, and so on, progressively. It is, therefore, impossible to judge even approximately the magnitude of the deviations by mere inspection. This can be done only with an ordinary chart which has a uniform vertical scale. The reason is that the former chart distorts the deviations, the latter does not. Such distorting charts are well known in geography. Mercator's projection of the earth globe onto a plane is an example. It likewise tends to exaggerate the distances along the vertical lines (meridians) and the exaggeration is greater the nearer one approaches the poles. Such charts cannot be used for measuring or even estimating meridional distances or areas. They serve only one purpose, to represent certain courses of navigation by straight lines.

Likewise, a Log-Probability Chart should be used only to represent an exact ogive by a straight line but not to judge how the data fit it. It is impossible to achieve any reliable judgment by mere inspection of

such a graph. For instance, if the data do not appear to fit a straight line it is quite impossible to decide by mere inspection whether they fit a curve or, say, a series of segments of different straight lines. We shall return to this point in a later paper.

As an example of what may happen if a Log-Probability Chart is used to estimate data by mere inspection, we reproduce in Fig. 4 the data for the emulsion, Seed Lantern No. 6102 (see Figs. 1, 2, and 3). These data are denoted by crosses. We have also drawn a straight line such as an inexperienced observer would draw by inspection in order to fit the data, according to his opinion, as well as possible. There are nine points shown in Fig. 4 (the tenth point corresponds to 100 per cent and lies at infinity; it is therefore not visible on the chart). Of these points, numbers 1, 2, 3, and 4 are below the level 99 per cent and are also well above the level 1 per cent. The remaining points, 5, 6, 7, 8, and 9, are above 99 per cent. Therefore, their deviations are considerably and progressively exaggerated. They are also subject to the wide statistical fluctuations which occur with small class-frequencies. However, the inexperienced observer ignores all this and acts as though this were an ordinary problem of fitting a straight line to some experimental points on a chart with uniform scale. He will, therefore, draw the line so that it passes as near as possible to as many of these points as possible, leaving part of them on one side and part on the other side of his straight line. This is the straight line drawn in Fig. 4, and it will be seen that the inexperienced observer has decided to draw his line through points 2 and 3 and very near to point 6. In doing so, he by-passes number 1 at the right side. This choice of a straight line by mere inspection is, however, poor, as will be shown in a later paper. From what has already been said it is clear that the observer should have given more consideration to point 1 than to 6 and the rest of them, because the former point occurs at levels well below 99 per cent and therefore deviation is less exaggerated. As a matter of fact, points 1 to 4 all lie on a nearly perfect straight line which by-passes point 6 and the rest at the right and leaves only number 5 at its left. (See Figs. 6 and 7.) In spite of this one-sidedness, it is a much better fit than the line shown in Fig. 4 and gives, as we shall see in a later paper, a fit which is, according to statistical tests, much better than the fit of the line shown in Fig. 4.

This is only one example of what can happen by treating a Log-Normal Distribution merely by graphical analysis. It would be easy to give other examples where the mere inspection furnishes estimates still more at variance with the correct estimate so that the goodness of the fit is much worse. Obviously, there is not much reliability in a straight line obtained from a Log-Probability Chart by inspection. On the other hand, we shall require such a line as a rough orientation of the data and shall use it as the basis of a more accurate treatment.

It is impossible to discriminate in such an inspection between the individual data according to the degree of exaggeration and distortion of their representation on the chart. If it is desired to achieve from the graph a rough estimate of the straight line which is somewhat reliable, the following rules of thumb should be observed: (1) Disregard all data outside the limits 1 per cent and 99 per cent. (2) Of the remaining data, give preference to those which are nearest the 50 per cent level.

CURVE FITTING AND THE CALCULUS OF OBSERVATIONS

The correct way of fitting curves to data, especially in such cases as ours, is to apply algebraic and analytical methods instead of the graphical procedure. Moreover, the goodness of fit obtained must be tested by the modern methods of statistics. Unfortunately, "the methods of determining the goodness or adequacy of fit do not appear to be . . . well known to physicists (and chemists)" (29).

There are several reasons for this lack of familiarity. First, these methods have been developed only in recent times by such eminent statisticians as Karl Pearson, Ronald A. Fisher, and others. Therefore, they have only recently, if at all, been included in the program for training physicists and chemists. Second, and even more important, these methods belong not to statistics proper but to the so-called Calculus of Observations. There is an essential difference between statistics and the Calculus of Observations. Statistics operates *inductively* and uses *hypotheses* which are adopted tentatively and can easily be abandoned if they fail the test. This is quite adequate in the major field of application of modern statistics which concerns mostly the so-called *inexact* sciences as biology, sociology, psychophysics, etc. On the other hand, the Calculus of Observations, if applied to the so-called *exact* sciences as astronomy, physics, chemistry, etc., deals with the application and testing of *theories* which have been obtained by *deduction* and cannot therefore be abandoned without serious consequences.[6] Now, the statistical methods mentioned before have been developed by statisticians proceeding in the inductive way just explained. They have managed, however, to provide the inexact sciences with a degree of accuracy unheard of before and comparable to that obtained so far only by exact sciences. On the other hand, in the latter field the older methods were for a long time considered to be sufficient and the progress achieved by modern methods was ignored. The classical book of

[6] In our present problem, for instance, the pure statistician would be satisfied with regarding the Log-Normal Distribution of particle sizes as a *hypothesis* which merely describes, but does not explain, the facts. However, if that hypothesis fails only in a single example, he would be inclined to reject it. On the other hand, the physicist wants his laws to be based on a *theory* such as is furnished in our case by the Exponential Law of Growth. A single failure of such a law would not be regarded by him as sufficient reason to reject it, as well as the theory on which it is based. If repeated observations confirm a failure, he would first try to modify his theory. Only if this fails to remedy that situation would he abandon the theory.

Whittaker and Robinson (30) antedates of course these modern methods. Of modern monographs—there are no modern textbooks on the Calculus of Observations—we mention the excellent book, "On Statistical Adjustment of Data," by Deming (31); it makes, however, only slight allusions to some of these matters.

As a consequence, it happens not infrequently that experimental data in physics and chemistry are, at present, treated inadequately and so lead not only to an inaccurate but to a *wrong* interpretation which could have been avoided, if the better methods now used by the inexact sciences had been adopted.

Our present problem is just such a case. Therefore, it is most desirable to allot more space to the explanation of the new methods than would be necessary under normal conditions.

There is also a special reason for going into some detail about these statistical methods in the present paper. Our problem is that of fitting *frequency curves*, and this is a special case of curve fitting in general. The statistical methods can be applied immediately to the case of fitting a frequency curve. The fitting of a general curve, on the other hand, permits the application of these methods only if the so-called experimental error is known from previous experiments or from special assumptions. The present case affords, therefore, an excellent illustration of these statistical methods. For all these reasons they are dealt with here to some considerable extent.

MUELLER WEIGHTS AND URBAN WEIGHTS

The correct way of treating the problem has been, curiously enough, known for a long time in the field of psychophysics. Psychophysicists deal with the sigmoid curve of response of any sensory organ to stimuli. They originally used a Normal Distribution ogive to fit their observations. Only recently[7] has the recommendation of Galton been heeded and a Log-Normal distribution used instead. This makes no difference for the question now before us, since that question concerns only the probability scale of our charts which is common to both distributions.

The German psychophysicist, Mueller (32), was the first (in 1878) to remark that the deviations from the probability scale carry a different *weight* according to the level of probability at which they occur. Consider the formula,

$$P = \int_{-\infty}^{t} z\,dt, \tag{5}$$

where

$$z = \exp(-t^2/2)/\sqrt{2\pi},$$

[7] Guilford (33) (p. 207). The normal distribution and the Log-Normal Distribution are known as the Phi-Gamma and Phi-Log-Gamma hypothesis, respectively, in psychophysics.

which connects probability, P, and normal deviate, t. Any error in P, say, δP, causes an error in t, say, δt, such that

$$\delta P = z\delta t.$$

Hence, the errors in the variable, t, must be multiplied by the factors, z, in order to represent correctly the observed errors in probability.

These multipliers vary from one level to another level. At level $P = 50$ per cent, we have, from the Probability Tables, $z = 0.39894$; at level $P = 99$ per cent, z is only 0.02667; and at level $P = 99.5$ per cent, we have $z = 0.01446$. The ratios of the z's for the three levels are therefore about 28:2:1. It will be remembered that the Probability Charts have the normal deviate plotted along their vertical axis. The deviations shown are therefore deviations in t and have to be multiplied by the factors, z, in order to reduce them to probability deviations. This means that a deviation at a level of 99.5 per cent has to be taken at 1/28 of its magnitude if it is compared to a deviation at a level of 50 per cent. Likewise, a deviation at 99 per cent must be reduced to about 1/14 of its magnitude. Thus, the exaggeration due to the scale of the Probability Chart mentioned in a previous section is taken care of fully.

Obviously, under these circumstances, the graphical procedure is no longer possible. The deviations occurring at different levels cannot be appraised correctly by mere inspection. It is, therefore, imperative to abandon the graphical procedure and to replace it by an analytical method, as for instance, the method of least squares. In the method of least squares, it will be remembered, a sum over the squares of the deviations is used which is to be rendered a minimum. However, the deviations must be taken with their correct value. Hence, if deviations in t are used, they must be multiplied by the Mueller factors, z, to change them into the correct deviations in P. The squares of the deviations in t are therefore to be multiplied with the squares of these factors. In other words, *weighted* least squares must be used. The *weights* are the z^2 and they are known as *Mueller weights*.

The Mueller weights are, however, not the only weights to be applied to the observations. As a matter of fact, they were necessary because of the exaggerations due to the probability scale on a Probability Graph, which is actually a t-scale and not a P-scale. Let us, however, use an ordinary chart as in Fig. 3, where the ordinates are graded according to a true P-scale. (Of course, the ogive is there represented by a sigmoid curve and not by a straight line as in the Probability Chart, Fig. 4.) Surely we now need no longer apply Mueller weights. However, we still have to use weights as the psychophysicist, Urban (34), noted in 1910.[8]

[8] See also Guilford (Ref. 33), p. 175.

The reason is that the experimental error to be expected is not the same for the different levels of P. If we want to compare the errors, δP, occurring at different levels, we must therefore *standardize* them before doing so, that is, we must divide each error, δP, by the corresponding Standard Error, S.

Urban gave the following formula for the mean square error, that is, the square of S, to be expected at the level, P:

$$S^2 = PQ/N, \tag{6}$$

where

$$Q = 1 - P.$$

This formula is derived in Appendix III by the use of advanced mathematics. Its meaning can, however, be understood by a more elementary, although less rigorous, reasoning. Let us recall that P is the probability that the particles have sizes not exceeding (say) x. An error in P can arise is two ways: Either the particles are all smaller than x or some of them are larger than x.

The probability that a particle is smaller than x is P. The probability that a particle is larger than x is the complementary probability, $Q = 1 - P$. The mean-square error will therefore be proportional to the combined probability of these two events, which is given by the product of their probabilities, PQ. To illustrate this formula, consider, for instance, the beginning of the range, $x = 0$. There cannot be any error in P, since there all particles are larger than zero. We have there $P = 0$. Therefore, the product, PQ, equals zero and hence the error is also zero, as it should be. Furthermore, if x increases, then P increases and so increases the product, PQ, but it increases more slowly than P because Q decreases. The product, PQ, will therefore increase slowly to a maximum which occurs at x_{median}, where $P = Q = 1/2$. Afterwards, it will decrease, because the decrease of Q now outweighs the increase of P. Finally, at the end of the range, $x = \infty$, the error must vanish, since all sizes must be finite. This is confirmed by our formula, since $Q = 0$ at $x = \infty$; hence, the product, PQ, and the mean-square error vanish.

This argument explains the numerator in Eq. 6. The denominator is also easily understood. Obviously, it is not the same if we find a probability level, P, by counting and classifying (say) $N = 1000$ grains or from $N = 10{,}000$ grains. In the latter case, the errors will be smaller than in the former case. Consequently, the mean square error should be inversely proportional to N.

This is a very important rule not always heeded by observers. Not only should as many grains be used as possible, but it is imperative to give the *absolute* data and not to reduce the data to 1000 grains, as it has become customary. It will be seen from Eq. 6 *that the omission of N makes it impossible to judge adequately the reliability of the data.*

From what has gone before it is clear that different weights for the deviations, δP, in P must be used before the method of least squares is applied, even if an arithmetic chart (Fig. 3) is used which shows the actual δP. The sum to be minimized in such a case will have to be weighted, *viz.*,

$$\chi^2 = \sum(\delta P)^2/S^2, \tag{7}$$

where the summation extends over the different observations. This sum is called the Chi-square and is the sum of the *standardized* squares of the errors, δP. The reciprocal of S^2, *viz.*,

$$1/S^2 = N/PQ,$$

is called the *Urban weight* of the error, δP.

If we use a Probability Chart which shows the errors, δt, in the normal deviate, t, and not the errors, δP, themselves, we have to use the following form of the Chi-square

$$\chi^2 = \sum W(\delta t)^2, \tag{8}$$

where

$$W = z^2N/PQ$$

is called the combined *Mueller-Urban weight* of the error in t. The formula (8) is easily derived from formula (7) by substituting

$$\delta P = z\delta t.$$

Therefore, only after weighting as just prescribed are we able to judge correctly the deviations shown on a Probability Chart and to avoid mistakes due either to the distortion by the scale or to the difference in the inherent accuracies of the different levels in P. Obviously such a weighting cannot be done in the process of inspection and this is why the graphical procedure fails.

Occasionally, some authors have used the least-squares method instead of the graphical procedure, but without these weights. This also cannot lead to correct results.

THE STANDARD ERROR BAND

Returning now to our problem of curve fitting, we start with the Log-Normal Ogive as drawn in Fig. 4 and determine intercept a and slope b from the graph, as described in Appendix II. This will furnish us the normal deviate from

$$t = a + b \ln x,$$

from which, using Probability Tables, we compute the corresponding values of the probability. To distinguish these values from the given data, we write these computed values with a prime, that is, t', P'. The data will be denoted by t, P.

The errors are then defined as the differences,

$$\delta P = P - P'.$$

These errors can be compared graphically with their respective standard errors by introducing into the graph the so-called *Standard Error Band*. It is limited by two lines which are given by

$$P' \pm S',$$

where

$$S' = \sqrt{P'Q'/N}$$

is the standard error according to Eq. 6 but referred now to the *computed* value of probability, P', instead of the observed value, P. In this way we obtain two curved lines which accompany the assumed ogive at both sides. They form between themselves the error band, one being its upper and the other its lower limit. This is shown in Fig. 5, which otherwise is a repetition of Fig. 4. The error band seems to widen towards the ends of the chart. This is a deception due to the distorting scale. If we had used an arithmetic chart, as in Fig. 3, the error band would appear to be the widest at the level 50 per cent and to become narrower and narrower towards the ends.

We now have a means of judging *approximately* the actual errors by mere inspection. We compare the vertical distances of the data from the ogive line at the level, P', with the width of the upper or lower half of the error band, as the case may be. This width is a measure of the standard error, S', and we can so visually determine whether the actual error is larger than the standard error or smaller, and even estimate how much. This gives us a rough idea about the value of the Chi-square in Eq. 7 or 8,

$$\chi'^2 = \Sigma(\delta P')^2/S'^2 = \Sigma W'(\delta t')^2$$

(the primes refer to the assumed ogive).

The value of the Chi-square which results from the ogive assumed in Figs. 4 and 5 is comparatively large. Consider only the points, numbers 1–5, and disregard points, numbers 6–9 (and number 10 at infinity), since they correspond to small class-frequencies and are therefore affected by irregularities. (Their treatment will be discussed in a later paper.) Of these points, numbers 1 and 5 are outside the error band and therefore contribute to the Chi-square, each more than unity. It will be remembered that the line was drawn by an inexperienced observer who tried to include as many points as possible in the ogive drawn and therefore had to by-pass numbers 1 and 5 in favor of the points, numbers 6, 7, etc.

It can now also be seen that this Chi-square cannot be the last word in curve fitting, that is, it is *not* the *minimum* value for which one should strive in curve fitting. It is easy to see how we can improve the fit by

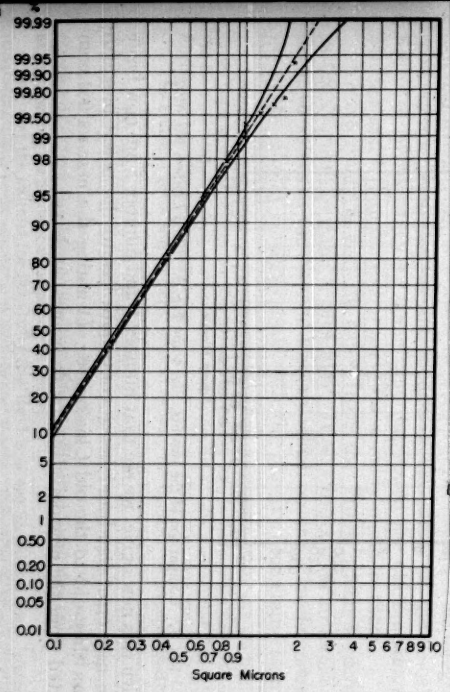

FIG. 5. The standard error band for Fig. 4. (Seed Lantern No. 6102.)

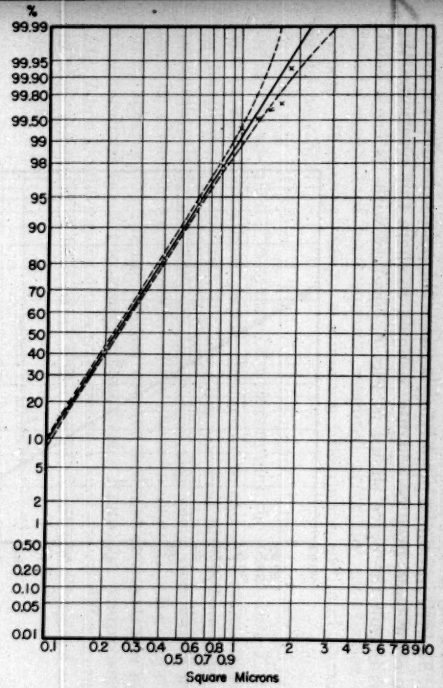

FIG. 6. A better fit than Figs. 4 and 5 obtained by using weighted least squares. (Seed Lantern No. 6102.)

reducing the Chi-square. Let us rotate counter-clockwise the ogive of Fig. 5 a little, around a pivot lying between the points, numbers 2 and 3. The error band may be roughly assumed to share this rotation. We thus obtain the ogive shown in Fig. 6, together with its error band.

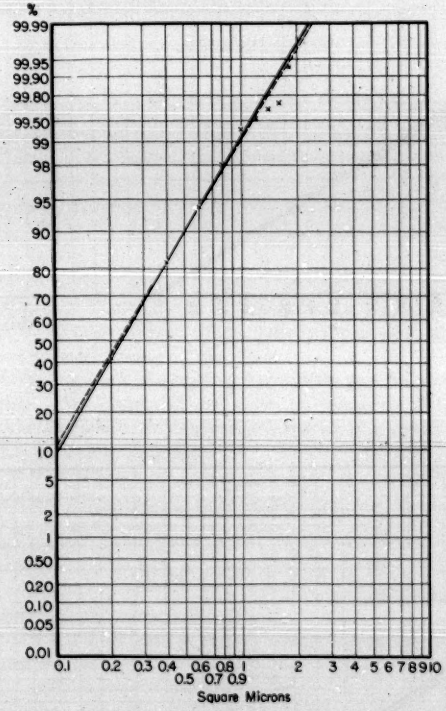

FIG. 7. Comparison of the two fits, Fig. 4 and Fig. 5. (Seed Lantern No. 6102.)

With this new ogive, we see that number 1 and number 5 are now inside the new error band and hence contribute less to the new Chi-square than they did to the old Chi-square. Denoting the new values computed from the new ogive by double primes, we have

$$\chi''^2 < \chi'^2.$$

This relation we can see now by mere inspection. What we cannot see is that the new ogive is the best fit, inasmuch as it renders χ''^2 a minimum. This will be shown in a subsequent paper.

In Fig. 7 we have drawn both ogives: First, the broken line, that is, the line assumed in Figs. 4 and 5, which we shall regard as a first approximation and which corresponds to a larger Chi-square, χ^2; second, the full line or second approximation shown in Fig. 6, which was computed by the method of least squares weighted according to Mueller and Urban. It corresponds to a minimum Chi-square, χ'^2, and therefore is the best fit.

One can introduce a numerical criterion of the *goodness of fit* by using the so-called *Chi-square Distribution* and the *Chi-square Test*, as will be shown in a later paper. Suffice it to say here that a certain probability is linked with the fit. For the first fit (broken line), the probability is 45 per cent; for the second (full line), it is 85 per cent, that is, about twice as much. This shows numerically the improvement achieved by the use of least squares and also demonstrates again the unreliability of the graphical method. From the comparison in Fig. 7 one would hardly guess that the two lines shown differ so much in their goodness of fit. More will be said about the Standard Error in a later paper.[9]

PROBIT ANALYSIS

The Mueller-Urban weights discussed earlier are mentioned in most modern textbooks on statistics only cursorily and under another name, *viz.*, the *standard error of a quantile* (Galton called our percentage, *P*, the "quantile"). By rights they belong in one of the several chapters on Curve Fitting, that is, Fitting of Frequency Curves. The foregoing remarks are illustrated by the modern development of the so-called Probit Analysis which for some time duplicated the older findings of Mueller and Urban instead of using these ready-made methods.

Probit Analysis gets its name from the Probit which is defined as the Normal Deviate, *t*, plus five. It was introduced to avoid the negative values of the Normal Deviate which occur for levels under 50 per cent. Probit Analysis deals with the toxicity of certain chemical

[9] Note that the Standard Error is different from the Probable Error and the Standard Error Band is different from the Confidence Band used, e.g., in Industrial Quality Control. We shall use neither of these concepts because they are linked with the so-called "Probability Limits," whereas the Standard Error is not.

The Probable Error was formerly used to indicate the limits of the experimental error. It is now being replaced for that purpose by the Standard Error for several reasons, one of which is that mentioned above. It goes without saying that no numerical value is complete without the ± limits of the Standard Error. These should never be omitted whenever, for example, a constant is determined from observations. Moreover, if two values of such a constant differ from each other by less than twice the Standard Error, they can be regarded as equal within the limits of the experimental error.

Note that the width of our Standard Error Band is ± one Standard Error. It is sometimes taken equal to ± two or ± three Standard Errors. Especially in Shewart's Control Chart in Industrial Quality Control does the "Three Sigma" Rule prevail. (See also Deming, Ref. 31, p. 171.) In our case, the width of ± one Standard Error is a "*must*," since the Band illustrates the Chi-square which, by equation (7), refers to *one* Standard Error.

poisons used as insecticides. The problem is to determine the degree of toxicity for increasing concentration of the insecticides. This is done by plotting the percentage of insects killed against the dosage. Obviously, different samples of insects must be used for each dosage. It is assumed that they all belong to the same population, that is to say, behave similarly towards that poison.

The plot mentioned furnishes a sigmoid response curve. It was found by Gaddum (35) that this curve is a Log-Normal Ogive. These findings were improved and analyzed statistically by Bliss (36), with the help of the eminent British statistician, Ronald A. Fisher (37). Since then, Probit Analysis has been treated in several books (Finney (38), Mather (39)) and tables (40) and has found wide application in biochemistry.

It is noteworthy that all these investigators did not use graphical methods such as the Probability Charts used in Size-Frequency Analysis. Instead, they used algebraic methods, although the subject would seem to invite graphical methods.

PROBIT ANALYSIS AND SIZE-FREQUENCY ANALYSIS

At the first glance, there seems to be much similarity between the problems of Probit Analysis and of psychophysics on one hand and Size-Frequency Analysis on the other hand. All three of them use the Log-Normal Distribution, especially its ogive, yet there is a fundamental difference.

In Probit Analysis and in psychophysics, one does not deal with *classification* of the subjects into different groups according to some characteristic, as is done in Size-Frequency Analysis. This is simply not possible, because there is no way to assay *quantitatively* the *response* of the subjects to the various dosages of poison or the intensities of the stimuli, respectively. One can only test whether or not an insect survives a dosage, or whether or not one subject finds a sensation greater than another. This is called the *quantal* response; instead of asking "how much?," one has to ask "yes or no?"; instead of a *classification*, one has a mere *dichotomy*.

In Probit Analysis and in psychophysics, one has therefore to start a new series of experiments for each value of the independent variable (dosage or stimulus). For instance, instead of using a single sample of, say, 100 insects and classifying each insect according to its degree of sensitivity to poison, many such samples are used, each for one value of the dosage, and the percentage of kill is determined for each dosage. All these samples must be *homogeneous*, that is, all the insects must belong statistically to the same population (behave similarly towards the poison). If that condition is satisfied, an ogive can be built up from many samples. The cumulative frequency of all insects killed by one dosage, x, is obviously the sum of all class-frequencies of insects

which have died from any dosage not larger than x. In this manner, a cumulative frequency curve is built up which represents the behavior of the insects for various degrees of dosage.

In Size-Frequency Analysis, a *single* sample is used and is classified. We are at liberty to consider the cumulative frequency curve (see Fig. 3) *or* the non-cumulative curve (see Fig. 2). If we use the former, we cannot, however, use the method of Probit Analysis. The cumulative frequencies of Size-Frequency Analysis are all from the same sample and are therefore not independent of one another. Nor are the errors of observation independent. The independency of the errors is, however, a fundamental condition which has to be satisfied before any least-squares method can be applied. Hence, we have to devise another method different from Probit Analysis.

The method which suggests itself is the analysis of the non-cumulated class-frequencies. They are not quite independent of one another (as we shall see in a later paper). Yet there is a method which takes care of their interdependence and it is a complete analogue of the method of least squares, although by no means identical with it. It is the method of the *Minimum Chi-square*. This Chi-square uses other weights instead of the Mueller-Urban weights, as we shall see in a later paper. On the other hand, it permits the immediate application of the Chi-square test mentioned above, and so enables us to control the improvement of the fit with each successive step of approximation.

It is worth-while to note here that Loveland and Trivelli (23) also used the non-cumulative frequency curve besides the cumulative ogive. They used again a graphical method and a special (logarithmic) chart, which are subject to the same objection as the use of the Log-Probability Chart. At any rate, this shows that *both* kinds of curves are used for the analysis of size-frequency which implies, as already mentioned, that Kapteyn's "Normalization" is successful only with the Log-Normal Distribution in our field.

HOMOGENEITY

In what has gone before we have mentioned homogeneity of the samples as a condition necessary before analysis can be applied. In conclusion, we shall add a few comments which are very important for the practical treatment of the subject.

Two samples shall be regarded as homogeneous or belonging to the same population if each obeys a Log-Normal Distribution and the parameters, a, b, of the two distributions are equal within the limits of the experimental or standard error. If these parameters are, however, different from each other, their differences exceeding those limits, the two samples shall be regarded as heterogeneous and as belonging to different populations.

This has an immediate application to a procedure practiced by some of the earlier observers in Size-Frequency Analysis. They used to

combine several samples of allegedly the same type (emulsions of the same kind, sand from the same material, etc.) and to compute the frequencies from an arithmetic average of these samples. This was done in the belief that the accuracy of the findings would be increased, owing to the larger number of observations used in that way. Actually, as can easily be shown, in most cases the accuracy decreased. The reason is that these samples belonged to different populations. For instance, the various emulsions of the same kind had been formed under not quite identical conditions and times and so differed, for example, in the values of the medians (that is, in the values of their parameters, a). Obviously, this is not known *a priori;* it can only be found out *a posteriori*, after performing a separate analysis of each sample, which, of course, is the only right way to deal with these samples. To increase the accuracy of the analysis, as many grains as possible should be used but they should be taken from the same emulsion and should all of them have been grown under identical conditions.

The foregoing remark refers to a kind of heterogeneity which is slight since it is due to differences in the values of the parameter, a. These differences are usually small and are caused by the deviations in the time at which the growth started. There is another kind of heterogeneity which is considerable. It is due to differences in the parameter, b, and is caused by different rates of growth. It is likely that this difference is inherent in the grains from their nuclear state and may be due to a preference of certain directions of growth. (It will be remembered that growth is anisotropic.) If two such different groups of nuclei are present in the emulsion from the beginning, they constitute two *entirely different populations*. Accordingly, they each obey a different Log-Normal Distribution which is independent of the other. Their superposition gives rise to what will be called a *compound* Log-Normal Distribution.

APPENDIX I

The Log-Normal Transformation

Obviously, the "Normalization" must transform both kinds of curves, the frequency curve and the ogive, into Normal Curves, since both are used in the analysis of the size distribution and it would not be acceptable if only one of them could be normalized.

Let

$$P = F(x)$$

be the probability curve (ogive) in the variable, x. The frequency curve will be given by

$$\frac{dP}{dx} = \frac{dF(x)}{dx}.$$

Now, assume that

$$t \simeq T(x)$$

is a normalizing transformation. Since the curves in x are skewed, the transformation must give different values for the median and the mode of x.

Let $T(x)$ be so chosen that the value $t = 0$ corresponds to the median of x and the value $t = -1/b$ corresponds to the mode of x. Here, b is some positive constant (the median is larger than the mode, as already mentioned).

Then, P as a function of t is normal and therefore necessarily of the form

$$P = \int_{-\infty}^{t} \exp(-t^2/2)dt/\sqrt{2\pi}.$$

By substituting, we can write

$$P = \int_{-\infty}^{T(x)} \exp(-t^2/2)dt/\sqrt{2\pi} = F(x);$$

hence, the frequency curve has the equation

$$\frac{dP}{dx} = \frac{dF(x)}{dx} = \exp(-T^2/2)\frac{dT}{dx}/\sqrt{2\pi}. \qquad (I)$$

According to what has been said above, this curve must also be normal and therefore of the form

$$C \exp(-[T + 1/b]^2)/\sqrt{2\pi}, \qquad (II)$$

where C is some constant which can easily be found from the total area below the frequency curve, which must be unity. By equating both expressions (I) and (II), we have

$$\frac{dT}{dx} = C \exp(-1/2b^2) \exp(-T/b),$$

or integrated

$$\exp(T/b) = C \exp(-1/2b^2)x/b.$$

(We have omitted an integration constant in x.[10]) This equation can be written

$$T = b \ln(C \exp[-1/2b^2]/b) + b \ln x = a + b \ln x,$$

where we have introduced a new constant instead of C, *viz.*,

$$a = b \ln(C \exp[-1/2b^2]/b),$$

which gives

$$C = b \exp\left(\frac{a}{b} + \frac{1}{2b^2}\right). \qquad (9)$$

[10] If the integration constant is included, we have to replace x by (say) $x - x_0$, which leads to a Log-Normal Distribution with a finite lower limit, discussed in the text with Eqs. 1b, 2b, and 3b.

We have therefore proved that the only transformation which normalizes ogive and frequency curve simultaneously is the logarithmic transformation, as in Eq. 3 of the text:

$$T = a + b \ln x.$$

Incidentally, we have obtained another form for the Log-Normal Frequency Curve,

$$C \exp [-(a + 1/b + b \ln x)^2/2]/\sqrt{2\pi},$$

where C has the value given by Eq. 9. This form will help in computing the frequency curve in practical applications. It gives the *mode* by

$$a + 1/b + b \ln x_{\text{mode}} = 0 \qquad \text{or} \qquad \ln x_{\text{mode}} = -a/b - 1/b^2.$$

The *median* is given, from Eq. 3, by

$$a + b \ln x_{\text{median}} = 0 \qquad \text{or} \qquad \ln x_{\text{median}} = -a/b.$$

Note that the expression $a + 1/b$ in the equation for the mode was denoted by a_1 in the text, Eq. 3.

It can easily be shown that the *arithmetic mean* is given by

$$\ln x_{\text{mean}} = -a/b + 1/2b^2.$$

Also, the mean square deviation around the mean is given by

$$\sigma^2 = (x_{\text{mean}})^2[\exp (1/b^2) - 1].$$

The *standard deviation*, σ, is the square root of this expression. Karl Pearson's *coefficient of variation* is given by the ratio

$$\sigma/x_{\text{mean}} = \sqrt{\exp (1/b^2) - 1}.$$

Hence, it is obvious that $1/b$ is a measure of *dispersion*, b is a measure of *uniformity*, and a/b is a measure of *location* for the Log-Normal distribution.

APPENDIX II

The Log-Probability Chart

If a logarithmic Probability Chart is to be used for quantitative analysis, it must have the same absolute scale along both axes. This is not the case with the commercial charts. The special chart reproduced in Fig. 4 in a reduced size (original dimensions 11 in. by 16½ in.) was produced by the Codex Book Company for the purpose of the present paper. It is so designed that the scale of the Normal Deviate, t, has the same absolute gradation as the logarithmic scale. For the latter, we take natural logarithms. The equation of growth imposes the use of the natural logarithm;[11] if the common logarithms were used as some authors did, we would have to carry, quite unnecessarily, in all

[11] Only the function e^x is reproduced by differentiation.

computations a factor of 2.30259. Tables of Natural Logarithms (41) are now available with sufficient range and number of decimals. If these tables do not suffice, the use of an adequate Table of Common Logarithms (42) and a multiplication machine is recommended. If we use natural logarithms, we have the further advantage that the slope, b, of the Log-Normal Ogive is of the order of unity, which is always best for accuracy. With common logarithms, the slope is 2.30259 times steeper.

Only with such a special chart can the slope be measured directly by a gradient meter (Gradient Meter of the Eastman Kodak Company or the Tangent Meter of Bausch and Lomb Optical Company). With the commercial charts, the slope obtained in such a manner would have to be reduced by a factor that depends upon the ratio of the absolute scale along the two axes, which is usually not accurately known.

Furthermore, the special chart is so designed that it permits accurate plotting of all percentages. It will be recalled that underlying the percentage probability scale is a scale of the Normal Deviate, t. Now, the absolute scale of the t is such that one unit of t on the original chart is reproduced as 50 mm. Hence, to plot a percentage, say, the datum for point No. 2 in Fig. 4, which is

$$P = 0.81553,$$

we look up the Normal Deviate which corresponds to it in the probability tables and find

$$t = +0.899.$$

This number multiplied by 50 gives us

$$+44.95 \text{ mm.}$$

as the ordinate, drawn upwards from the midline (50 per cent) at the abscissa of No. 2 ($x = 0.4$). This is done by using a micrometer caliper gauge, manufactured by the L. S. Starrett Company, which permits the measurement of a distance to 1/100 of a millimeter.

Likewise, on the other axis, one unit of the natural logarithm on the original chart is represented by a length of 50 mm. It is, therefore, possible, using a table of natural logarithms, to plot accurately any abscissa, x, which does not coincide with the divisions given by the chart.

Also, it is possible to read percentages, P, from the chart by reversing the procedure given. The same holds for the sizes, x.

With such a special chart, it is moreover possible to construct, without computation, the mode and the mean of a Log-Normal Distribution. In Fig. 8, a Log-Normal Ogive is represented by the straight line, DAH. It intercepts the midline (50 per cent) at the point A, which is the median and encloses the angle, β, such that

$$\tan \beta = b.$$

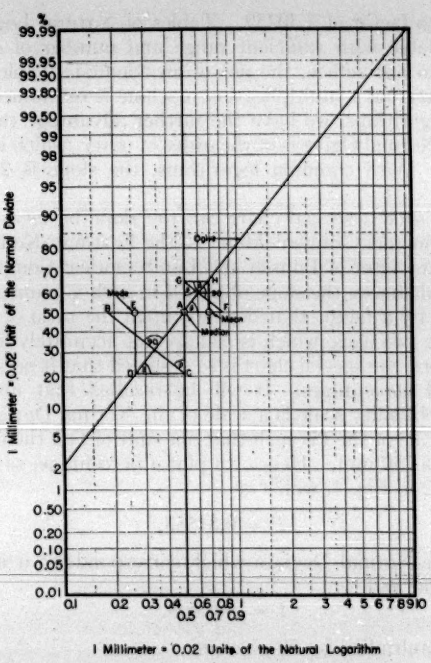

FIG. 8. Construction of mode and mean. (AB = 50 mm., AF = 25 mm.)

Let us measure a distance, AB, on the midline to the left of the median such that $AB = 50$ mm. or 1 unit of the natural logarithm. Then BC is drawn perpendicularly to the ogive. In the right triangle, ABC, the angle at C equals β. Therefore,

$$AC = AB \cot \beta = 1/b.$$

A horizontal line, CD, is drawn. In the right triangle, ACD, the angle at D equals β; hence

$$CD = AC \cot \beta = 1/b^2;$$

D is projected on the midline and the point E is the mode, because (Appendix I)

$$\ln x_{\text{mode}} = \ln x_{\text{median}} - 1/b^2.$$

In the same manner, the mean can be found by starting with a point, F, at the right of the median such that $AF = 25$ mm. or $1/2$ the unit of the natural logarithm.

Drawing a line, GF, perpendicular to the ogive furnishes

$$AG = AF \cot \beta = 1/2b,$$

and drawing the horizontal, GH, gives

$$GH = AG \cot \beta = 1/2b^2.$$

Projecting H onto the midline furnishes the point, I, which is the mean, because (see Appendix I)

$$\ln x_{\text{mode}} = \ln x_{\text{median}} + 1/2b^2.$$

APPENDIX III

Bernouilli's Binomial Law and De Moivre's Limiting Normal Form of Bernouilli's Law

To find the experimental error of any observation, one must have numerous repetitions of that observation. In our case, where the observation is a frequency, it is possible to predict the result of such repetitions.

Consider a cumulative frequency, F, corresponding to a size, x. If we repeat the observations, we shall get different values of F. All these values must, obviously, be contained between the limits, zero and N, where N is the total number of grains. Assume that we are given a probability, P', that we find a particle which has a size not larger than x. All such particles contribute to the cumulative frequency, F. What is the compound probability of finding just F particles corresponding to the size, x?

The answer is given by James Bernouilli's Binomial Law which follows from combinatorics. If $Q' = 1 - P'$ is the complementary probability to P', that compound probability is given by

$$\frac{N!}{F!(N-F)!} P'^F Q'^{N-F}.$$

We have so the distribution for the observations, F. More specifically, we can find their mean value, $\bar{F}$, and their mean-square deviation around that mean. The mean value is obtained by summing over all possible values of F, each being multiplied by its relative frequency, that is, the compound probability just given:

$$\bar{F} = \sum_{F=0}^{N} F \times \frac{N!}{F!(N-F)!} P'^F Q'^{N-F}.$$

The result is

$$\bar{F} = NP'.$$

Likewise, we find for the mean-square deviation around zero,

$$\overline{F^2} = \sum_{F=0}^{N} F^2 \times \frac{N!}{F!(N-F)!} P'^F Q'^{N-F} = N(N-1)P'^2 + NP'.$$

The mean-square deviation around the mean is, therefore,

$$\overline{(F - \bar{F})^2} = NP'Q'.$$

Dividing by N^2, we have finally, for the mean-square deviation of the probability, $P = F/N$:

$$S^2 = \frac{P'Q'}{N},$$

which is Urban's formula, Eq. 6 of the text.

The above derivation presumes only that we have given a probability, P', for the single event that a particle has a size not larger than x, and that this probability remains constant throughout. Such a probability is given by the Log-Normal distribution which we have assumed to govern the distribution of particle sizes. Let F' be the cumulative frequency which, according to this theory, corresponds to the size, x. Then we have

$$P' = F'/N$$

(from the frequency definition of probability).

At the same time we have obtained now a law of errors for our observed frequencies, F. Substitute for that purpose

$$F = F' + \delta,$$

where δ is the error in the cumulative frequency, F, into the compound probability given above. We obtain

$$\frac{N!}{(F' + \delta)!(N - F' - \delta)!} (F'/N)^{F'+\delta}(1 - F'/N)^{N-F'-\delta}.$$

Here, $F' + \delta$ is supposed to be an integer, so that we have a discontinuous distribution law for the errors, δ. Moreover, no error can be smaller than $-F'$, nor can it be larger than $N - F'$. The distribution of our errors is therefore limited both ways. Finally, it is not symmetric with respect to the mean which is given by $\delta = 0$ or $F = \bar{F} = F'$. Our error law is, therefore, quite different from the Normal law of errors. However, under certain assumptions, as De Moivre has shown, it is possible to approximate our error law by the Normal law.

These assumptions are: (1) The three quantities, N, F', $N - F'$, are large; (2) The errors, δ, are small. Under these conditions one may replace the binomial coefficient in the above expression by its Stirling approximation. Furthermore, one may develop the resulting expression

logarithmically into a Taylor series and neglect terms of higher order than the second. The result is

$$\text{Const. exp. } [-\delta^2/2F'(1 - F'/N)],$$

which has the form of the Normal law. The mean is again $\delta = 0$ and the mean-square deviation is again $S^2 = F'(1 - F'/N)$, as in Urban's formula. The errors, δ, may now be assumed to be distributed continuously. Moreover, the distribution is unlimited both ways and is symmetric.

However, it should be well borne in mind that this Normal law is only an approximation. For instance, it does no longer hold, even approximately, when one of the quantities, F', or $N - F'$ is small. This will happen at the beginning and at the end of the range of the observations. We shall see in a later paper the consequences engendered by this.

In conclusion, we point out again that the mean-square deviation, $S^2 = F'(1 - F'/N)$, depends upon F' and therefore varies according to the size, x, considered. The observations must be given different weights according to x. No unweighted least squares must be used.

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Beta Gauge Thickness (*Instrument Practice*, Vol. 4, No. 10).—Now on test in a Lancashire textile mill is the beta thickness gauge, an instrument which had its first public showing in Manchester recently, prior to going into the mill.

The instrument owes its development directly to the work going on at Harwell since, of course, the atomic research work there has resulted in the commerical development of beta rays.

To produce a yarn of even diameter, the cotton spinner has to pass the sliver and roving through a succession of doubling operations in order to even out the irregularities of the sliver. This, naturally, is a costly process and involves repeated handling of the material. If it were possible to produce a perfectly even sliver and maintain its evenness throughout the manufacturing processes, many of the doubling operations would be eliminated. This would have the effect of reducing the cost of production and also in shortening the manufacturing process.

In the past, the loose and spongy nature of the sliver has contributed to the difficulty of ascertaining a satisfactory method of measuring the variations which occur.

It is impossible to use any mechanical means for measuring it but the beta thickness gauge has provided the answer because it can measure the weight per unit length of the sliver without touching it.

The principle of beta rays is well known and requires no amplification here; the equipment comprises three main parts: the isotope, the ionization chamber situated below it and the amplifier unit and instruments.

Suffice to say that the beta rays are propelled from the isotope and the natural characteristic of the rays enables the amount of radiation passing through the material to be measured.

The leakage current is, of course, very minute, but by suitable electronic amplification it may be utilized to work an indicating instrument and/or recorder calibrated in degrees of thickness of weight per unit area, etc., and this can be used to operate servo devices for applying any necessary correction.

The instrument has been developed by the Baldwin Instrument Co., Ltd., Dartford, Kent, in collaboration with F. C. Robinson & Partners, Ltd., 287, Deansgate, Manchester.

Television cameras may replace pilots in hazardous test flights under a system developed by the Air Materiel Command at Dayton, Ohio. Operated from the ground by remote control, the planes would be tested to the limit at critical speeds and stresses, with cameras focused on the instrument panel and recording data on screens set up below. Pilots will still be used on routine flights, but unusually dangerous tests may be run without risking human lives. The method may be effective even on very high speed planes. (Details available.)

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

PRECISION THERMOSTAT FOR HIGH TEMPERATURES

An improved precision thermostat developed by Dr. Wm. R. Eubank at the National Bureau of Standards provides smooth, continuous control of an electric furnace within a very small range at temperatures between 1000° and 1550° C. The device is of the type in which the furnace winding itself serves as the sensitive element, forming part of a bridge circuit for control of a thyratron tube. The thyratron circuit then acts as a continuously variable valve, allowing just enough current to reach the furnace to compensate for a given temperature fluctuation. The new thermostat is highly sensitive and independent of normal fluctuations in line voltage. As it is convenient to operate, easily adjusted for any temperature, and reliable for periods of continuous operation from a few minutes to several days, it should provide a useful research tool in such fields as metallurgy, ceramics, glass technology, and cement chemistry, where automatic control of furnace temperatures has long been a problem.

In exploring new compositions of glass, refractories, portland cement, and other industrial products at the National Bureau of Standards, systematic phase-equilibrium studies have been of great value. However, one important difficulty in this work has been the necessity of maintaining the specimen under study at a constant high temperature until a state of equilibrium is reached. Even small fluctuations in temperature may result in erroneous data, especially in multicomponent systems. In an effort to obtain better temperature regulation than was possible with regulators currently in use, the Bureau therefore undertook the development of an improved thyratron regulator for temperatures above 1000° C. The resulting circuit maintains furnace temperatures constant within $\pm$ 0.1° C. for several hours, and within $\pm$ 1.0° C. for several days.

In the Bureau's thermostat, the output of an a-c. bridge, one arm of which serves both as a resistance thermometer and as a heater element for the furnace, is amplified and applied to the control grid of a thyratron. When the resistance of the furnace winding changes slightly with temperature, the resulting unbalance of the bridge produces a difference in phase between the control voltage and the anode voltage of the thyratron. This phase difference determines the firing period of the thyratron, and the thyratron action is reflected to the bridge circuit through a

* Communicated by the Director.

transformer, thus controlling the amount of energy supplied to the furnace.[1]

Power is supplied to the bridge circuit by a constant voltage transformer. A variable autotransformer then reduces the voltage output of the voltage regulator to the value needed for a particular operating range. In series with the autotransformer primary is a variable resistance. This resistance is also connected in parallel with the 25-volt primary of a 250-volt-ampere transformer whose 100-volt secondary is in the plate circuit of the thyratron.

The thyratron controls the furnace current by changing the impedance of the primary winding of the transformer. If the phase of the control voltage is such that no plate current flows in the thyratron, the transformer is unloaded and its impedance is high. Under these conditions, a negligibly small magnetizing current flows in the primary. When the thyratron fires, however, the impedance of the primary becomes low relative to the variable resistance. The bridge current, and thus the current through the furnace, is thereby increased, nearly all of it passing through the primary of the transformer. Thus the transformer reflects the high-voltage, low-current characteristics of the thyratron switch into the power circuit as a low-voltage, heavy-current control.

The four arms of the a-c. bridge consist of the furnace winding (about 2 to 15 ohms), a 2-ohm resistor, a 2000-ohm resistor, and a variable resistance consisting of a 5000-ohm coil and a 10,000-ohm decade box in series. The resistance of the furnace winding, which consists of about 100 g. of 0.8-mm. platinum–20%-rhodium wire, changes rapidly with temperature above 1000° C. and except for aging effects, maintains a definite relation to the temperature. The high ratio between the resistances on the two sides of the bridge is possible because the amplifier is voltage-operated.

Since alternating current is used, both the reactive and resistive components of the bridge must balance. For this reason, the resistance coils in the bridge, except those of the furnace, are noninductively wound. The reactance of the furnace is then balanced out by adjusting the variable secondary of a mutual inductance located in the same arm of the bridge as the variable resistance. In order to use the mutual inductance with furnaces of various inductances, the primary winding is tapped at three points.

The amplifier has a gain of about 6000 and is of the conventional resistance-capacity coupled type. The thyratron tube employed is the sensitive, four-electrode 3D22. This tube, because of its inert gas content, has a control characteristic which is essentially independent of

[1] This type of circuit was first described by H. S. Roberts in "Temperature: its measurement and control in science and industry" (*Am. Inst. Phys.*), pp. 604–610, Reinhold Publishing Corp., New York (1941).

ambient temperature variations over a wide range. It also has such advantages as high current capacity, small preconduction or gas-leakage currents, low control-grid-to-anode capacitance, and low control-grid current.

In practice, the furnace is first brought to approximately the desired temperature by manipulation of the autotransformer. The variable bridge arm is then set at a value found by experiment to correspond to the temperature desired. During long periods of continuous operation the balancing resistance is usually adjusted at about three-day intervals to compensate for the drift in the resistance of the winding.

In several hundred short quenching esperiments at the National Bureau of Standards lasting from 30 minutes to an hour, average temperature variations with the thermostat corresponded to 1 or 2 microvolts in a platinum–platinum-rhodium thermocouple ($1\mu V = 0.084$ degree at 1300° C.). Dependability for long experiments was demonstrated in constant control of a quenching furnace at 1300° C. for more than a month and at 1350° for three weeks. The average of 20 consecutive fluctuations occurring over night and during week ends was less than 0.5° C.

An advantage in the use of the phase-controlled thyratron is gradual or proportional control as contrasted to on-off control. Over-shooting, which appears with on-off control, is greatly reduced. It is expected that the control circuit developed at the Bureau will find application in the regulation of other furnaces and in baths and heaters operating at lower temperatures. Heater elements, such as pure nickel or certain nickel-chromium alloys, which have sufficient change of resistance with temperature in the lower temperature ranges would, of course, be required in the latter applications.

THE FRANKLIN INSTITUTE

MEDAL DAY MEETING

WEDNESDAY, OCTOBER 18, 1950

The Annual Medal Day Exercises of The Franklin Institute were held on Wednesday, October 18, 1950, in the Benjamin Franklin Memorial Hall. Approximately 260 persons attended the dinner and the presentation of awards. Prior to the dinner, a reception in honor of the 1950 Medalists was held in the rooms adjoining Memorial Hall.

The Medal Day Exercises opened with the playing of the National Anthem by Mr. Guy Marriner, Director of Music at The Franklin Institute. Mr. Richard T. Nalle, President, presided at the ceremonies.

Following the dinner, Mr. Nalle introduced the Honorable Gene D. Smith, Secretary of the Commonwealth of Pennsylvania, who spoke briefly on Pennsylvania Week and expressed his gratitude to The Franklin Institute for its part in building a better Pennsylvania.

The Stated Monthly Meeting of The Franklin Institute was held in conjunction with the Medal Day Exercises. The minutes of the Stated Meeting in May were approved as published in the June issue of the JOURNAL. Mr. Nalle announced that Dr. Eugene Paul Wigner, recipient of the Franklin Medal, had been elected to honorary membership in the Institute and that his certificate of membership would be presented to him later in the evening.

The President then requested that Mr. Lewis P. Tabor, Chairman of the Committeee on Science and the Arts, introduce the twenty-one former medalists who were present.

Dr. Henry B. Allen was then called upon to give the Secretary's report on "The Franklin Institute in 1950."

Following Dr. Allen's report, the presentation ceremonies for the eleven medals awarded in 1950 were opened by a short message from the President, expressing regret that three of the medalists were unable to be present to receive their medals: Dr. Eugene P. Wigner, recipient of the Franklin Medal, was represented by his wife, Dr. Mary Wheeler Wigner; Dr. Basil F. J. Schonland, Elliott Cresson Medalist, was represented by Mr. D. R. Masson, Scientific Attache of the Embassy of Union of South Africa; and Mr. R. M. Dubois, Inter-Continental Equipment Corp., received the Frank P. Brown Medal for Monsieur Eugene Freyssinet.

At the conclusion of the presentation of the awards, Mr. Nalle introduced Dr. Mary Wheeler Wigner, who read the paper "The Role of Mathematical Methods in Physical Theories," which had been prepared by her husband. This paper, as well as the full proceedings of Medal Day, will be published in the December issue of the JOURNAL.

HENRY B. ALLEN,
Secretary

PROGRAM

Reception for Medalists.....THE HOSTESS COMMITTEE
Assembly in Franklin Hall.....Upon gong signal at seven-ten
National Anthem.....GUY MARRINER
Dinner.....MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin.....RICHARD T. NALLE, *President*
Greetings.....THE PRESIDENT
October Stated Meeting of The Institute.....THE PRESIDENT PRESIDING
Introduction of Former Medalists.....THE PRESIDENT
The Institute in 1950.....HENRY B. ALLEN, *Secretary*
Presentation of Awards.....THE PRESIDENT
Presentation of Dr. Wigner's paper on "The Role of Mathematical Methods in Physical Theories"
(Read by Dr. Mary Wheeler Wigner, in the absence of Dr. Eugene Wigner.)

MEDALISTS

To be Awarded a Potts Medal (1906)

MERLE A. TUVE
Director, Department of Terrestrial Magnetism
Carnegie Institution of Washington

To be Awarded a Henderson Medal (1924)

PAUL WALTER KIEFER
Chief Engineer, Equipment
New York Central System
New York, New York

To be Awarded Wetherill Medals (1925)

DONALD W. KERST
University of Illinois

KENNETH C. D. HICKMAN
Inventor
Rochester, New York

RUSSELL H. VARIAN
President, Varian Associates
San Carlos, California

SIGURD F. VARIAN
Director of Development Laboratory
Varian Associates
San Carlos, California

To be Awarded Brown Medals (1938)

EUGENE FREYSSINET
Société Technique pour l'Utilization de la Précontrainte
Paris, France

GUSTAVE MAGNEL
University of Ghent
Ghent, Belgium

To be Awarded a Clamer Medal (1943)

CHARLES SANBORN BARRETT
Institute for the Study of Metals
University of Chicago

To be Awarded a Cresson Medal (1848)

BASIL F. J. SCHONLAND
South African Council for Scientific and Industrial Research
Pretoria, South Africa

To be Awarded the Franklin Medal (1914) and a Certificate of Honorary Membership

EUGENE P. WIGNER
Thomas D. Jones Professor of Mathemetical Physics
Princeton University

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, October 11, 1950.)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 11, 1950.

MR. LEWIS P. TABOR in the Chair.

The following report was presented for final action:

No. 3201: Electrostatic Precipitator.

This report recommended the award of a John Price Wetherill Medal to Gaylord W. Penney, of Pittsburgh, Pennsylvania, "In consideration of his work in the development of a practical and commercially used electronic precipitator to remove particles of matter from ventilating air."

JOHN FRAZER,
Secretary to Committee.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* A New Principle in Two-Band Rotary-Beam Design.**[1]—A. M. PICHITINO.[2] In recent years the parasitic type of beam antenna array has been widely used in radio communication. The characteristics of this type of array—appreciable gain at low vertical angles of radiation, good azimuthal resolution or discrimination, wide coverage when rotated, compactness—are well known and have proved highly desirable. The usual approach to dual-band operation with this type of antenna has been to utilize separate antennas mounted upon a common rotatable structure. Previous articles[3,4,5] have described the use of relays and linear phasing sections to achieve dual-band operation with common radiating elements. These methods have disadvantages in that adjustments are difficult and tedious and that proper relays are difficult to obtain and maintain.

This paper describes a dual-band parasitic array using common radiating elements without the use of relays or linear phasing sections. Tuning of the system is easier, less time-consuming and more accurate than in the case of a single-band antenna. An improved tuning method for single-band antennas is also described.

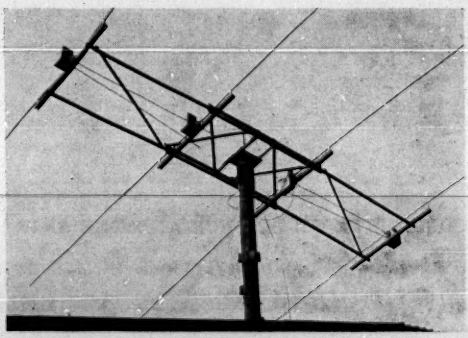

FIG. 1. Close-up view of the experimental antenna, showing the weatherproof enclosures for the networks and the extension rods for tuning the networks associated with the outside elements. The loop of 300-ohm line is aslo visible in this photograph.

THEORY

This system provides dual-band operation at maximum efficiency only when the frequencies are harmonically related at a frequency ratio of one to two (14 Mc. and 28 Mc., 7 Mc. and 14 Mc., etc.).

Basically, the array consists of a parasitic-type four-element antenna-driven element, two directors and a reflector—cut for the lower frequency using 0.1 λ spacing between all elements. All elements are insulated and open in the center.

The driven element is energized as a half-wave dipole on the lower frequency and as two half-waves collinear, in phase, on the higher frequency. The parasitic elements must be shorted at the center on the lower frequency and opened at the center on the higher frequency. This is done to provide the usual half-wave parasitic elements on the lower frequency to work in

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[1] Published in *QST*, October, 1948.
[2] Senior Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.
[3] J. D. KRAUS, "The Three-Band Rotary Antenna," *Radio*, February, 1940.
[4] J. A. MCCULLOUGH, "A Unique Five-Band Antenna System," *QST*, December, 1946.
[5] J. G. MARSHALL, "Matching the Antenna for Two-Band Operation," *QST*, September, 1945.

conjunction with the half-wave driven element, and to provide half-wave collinear parasitic elements that are properly phased to work with the higher-frequency collinear driven element.

In other words, the antenna may be described as a conventional four-element close-spaced (0.1 λ) beam on the lower frequency and as eight elements wide-spaced (0.2 λ) on the higher frequency, each of the collinear half-waves of the driven element being headed by two directors and backed by a reflector.

In view of the fact that the experimental antenna was designed and put into operation on 14 Mc. and 28 Mc., further discussion will use the terms "14 Mc." and "28 Mc." in place of "lower frequency" and "higher frequency," respectively.

IMPEDANCE MATCHING

The impedance of a center-fed dipole differs greatly from that of two collinear half-waves fed in the center, especially when, as in this case, parasitic elements are used. Experimental measurements indicate that the impedance at 14 Mc. is of the order of 5 ohms and at 28 Mc. is 4500 ohms with parasitic elements. A network, Fig. 2, was constructed with constants such that a common transmission line would be properly terminated at both frequencies. This network is inserted at the center of the driven element and results in an essentially flat line on both 14 and 28 Mc.

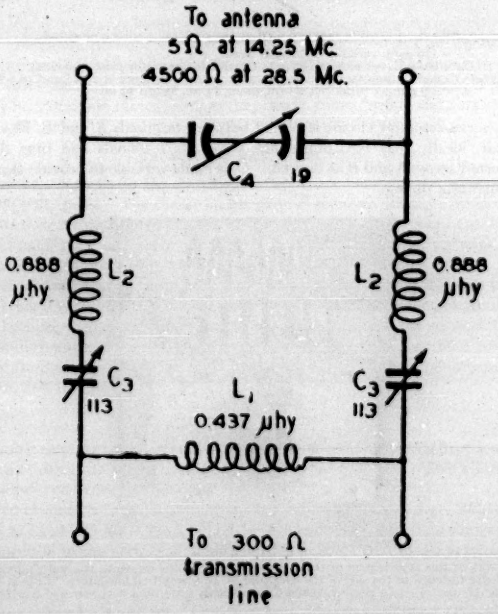

FIG. 2. Circuit of two-band matching network to couple a 300-ohm line to a 5-ohm load at 14.25 Mc. and to a 4500-ohm load at 28.5 Mc.

C_3 = 116-μμfd. variable, 3800 volts peak.
C_4 = 35-μμfd.-per-section, 4200 volts peak (Cardwell NP-35-ND).
L_1 = 0.437 μhy.; 4 turns No. 8 wire, diam. 1-9/16 inches, length 2½ inches.
L_2 = 0.888 μhy.; 6 turns No. 8 wire, diam. 1-9/16 inches, length 1-9/16 inches.

PARASITIC NETWORKS

As stated previously, it is necessary to short the parasitic elements electrically on 14 Mc. and to open them electrically on 28 Mc., in the center of the element. In the experimental antenna, which is centered on 14.25 Mc., the directors are 4 per cent shorter and the reflector 5 per cent longer than the driven element, which is 0.95 $\lambda/2$.

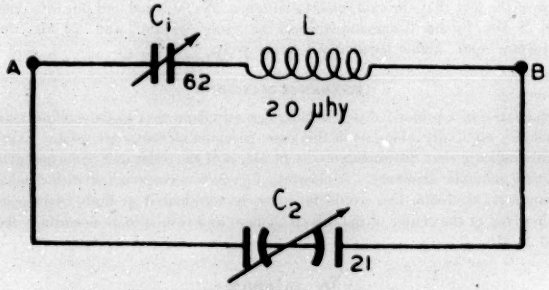

FIG. 3. Circuit of network used at the center of parasitic elements.

C_1 = 116-μμfd. variable, 3800 volts peak.
C_2 = 50-μμfd.-per-section (same as C_1 with center stator plate removed).
(Numbers C_1 and C_2 indicate calculated values to effect resonance at 14.25 and 28.5 Mc.)
L = 2.0 μhy.; 7 turns No. 8 wire, diam. 2½ in., length 4¼ in.

A 14 Mc. series-resonant circuit is placed between terminals A and B, Fig. 3. A series-resonant circuit, ideally, has zero impedance across its terminals and thus the element is electrically shorted across A and B as desired. C_1 is made variable to provide electrical adjustment and to facilitate tuning.

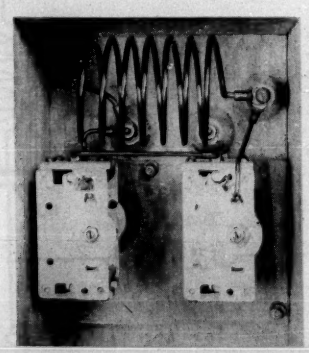

FIG. 4. A network of this type is used at the center of each parasitic element. C_1 and C_2 are mounted on a sheet of Lucite fastened to the side of the enclosure by ½-in. stand-off insulators. C_1 is at the right and C_2 at the left.

Since there is zero impedance across A and B at 14 Mc., any impedance may be placed across these terminals with no effect upon the tuning of the parasitic element. Capacitor C_2 (Fig. 3) is placed across A and B and is of such a value that at 28 Mc.,

$$X_L - X_{C_1} = X_{C_2}$$

When this equation is satisfied, it can be seen that C_2 in conjunction with C_1 and L provides a parallel-resonant circuit between A and B at 28 Mc. A parallel-resonant circuit, ideally, presents infinite impedance across the terminals and thus the element is electrically open at the center as desired. (In the practical case, the impedance is not infinite but is determined by the Q of the tuned circuit. With a good coil, having a Q of 200 or more, the impedance is sufficiently high between terminals A and B to insure proper operation of the parasitic elements.) C_2 is also made variable to provide for easy adjustment.

Capacitor C_3 has no effect upon 14 Mc. tuning as it is effectively shorted at this frequency. Capacitor C_1 does affect tuning at 28 Mc. because C_2 must be varied to effect parallel resonance with each change of capacitance of C_1. Therefore, to avoid interaction in tuning, it is necessary that the antenna system be tuned first as 14 Mc.

SUPPORTING STRUCTURE

The element-supporting structure is of conventional design and is constructed of wood. The center spacing between all elements is 0.1 λ (free space value) at 14.25 Mc., which is equal to 6 ft. 11 in. It is recommended that at least 4-in. insulators be used throughout because on 28 Mc. the center of each conductor is at a high-voltage point.

PARASITIC-NETWORK CONSTRUCTION

The parasitic networks are identical for each parasitic element and are shown schematically in Fig. 3. A completed network is shown in Fig. 4. The networks are enclosed in wooden boxes for weatherproofing as in the case of the matching network. The capacitors and inductor are insulated from the box, with the capacitor shafts extended (by use of flexible insulated couplers and $\frac{1}{4}$-in. aluminum tubing) so that they can be rotated while standing at the top of the supporting mast.

CAPACITORS AND INDUCTORS

All capacitors except C_4 (in the matching network) are basically the same—single section, 116 $\mu\mu$fd., 3800 volts peak. The center stator plate is removed (using a jeweler's saw) in the case of the parasitic-network shunt capacitors, C_3, which provides proper capacity and voltage breakdown (25 $\mu\mu$fd. max. and 7600 volts peak) as a split-stator capacitor with rotor floating.

All inductors are wound of No. 8 solid copper wire and are self-supporting. The parasitic-network inductors can be wound using the specified turns and sizes as they can be pulled apart or pushed together if necessary. However, the matching-network inductors should be accurately measured, because even though the variable capacitors will tune the network through a wide range, the standing-wave ratio will not be as favorable if the inductors differ from those specified. The resultant s.w.r. with correct inductors, as measured on the experimental antenna, was 1.08 on 14 Mc. and 1.34 on 28 Mc.

ELEMENT LENGTHS

It is desirable that the parasitic networks perform only the switching function and that they be required to compensate for element length as little as possible. This allows full element lengths and normal current distribution.

The driven element is 32 ft. 10 in. long (with a 1-in. gap in the center) which length corresponds to 0.95 $\lambda/2$ at 14.25 Mc. Each side is 16 ft. 5 in. in length with no compensation for the length of connection to the network. This length is in effect part of the network and is taken care of in the network tuning.

The directors are made 4 per cent shorter than the driven element or 31 ft. 6 in. (with a 1-in. gap in the center). Each side is 15 ft. 6 in. in length with 3 in. allowed each side for connection to the actual network.

The reflector is made 5 per cent longer than the driven element or 34 ft. 6 in. (again using a 1-in. gap in the center). Each side is 17 ft. in length with 3 in. allowed for connections as in the case of the directors.

TUNING PROCEDURES

The antenna system may be tuned conventionally by supplying power to it and adjusting the variable capacitors to obtain the desired reading on a remotely located field strength meter or by tuning for maximum received signal. The antenna matching network should be adjusted to obtain minimum s. w. r. on the transmission line.

SINGLE-BAND TUNING

It is obvious that the series-resonant method of tuning may be applied to single-band antennas. It is necessary only to provide a series-resonant circuit to tune to the operating frequency. Actual field-intensity measurements show that there is no measurable difference between a solid parasitic conductor and an element using a series-resonant network. Although it appears that the I^2R losses should be greater with series-resonant tuning, it does not show up in field-intensity measurements. This type of tuning makes it much simpler to tune a single-band antenna regardless of spacing or height above effective ground.

RESULTS

It is believed that this type of tuning of either single-band or multi-band antennas is more effective than any other method and facilitates accurate tuning to a considerable degree.

Services which have used this system successfully are amateur, military, television, and mobile.

MUSEUM

How deep is an oil well? That is a question frequently asked by visitors to the Chemistry Section of the Institute's museum. They do not display any surprise when told that the deepest well in California goes down 15,000 feet. Surprise concerning the petroleum industry appears to have died away. The ordinary man in the street has been surprised by the petroleum industry so often that he has come to expect the unexpected from it.

Petroleum derivatives are now so well recognized that there is a revival of interest in petroleum itself, as apart from its better known gasoline. However, visitors to the elaborate Sun Oil Company's exhibit in the museum still display a deep interest in the refining of petroleum because this is the source of their gasoline. The additional derivatives earn only a subsidiary interest. This particular exhibit is ideally designed for the instruction of the layman.

It comprises two units, one to illustrate oil field operations and the other to demonstrate the refining process. The oil field model shows a rotary drilling rig at work, a pumping well, and a flowing well. Underneath are the geological strata of a characteristic oil field.

The model of the refinery units is much more elaborate. There are three sections to this model: primary distillation, catalytic cracking, and the mercury vapor process for lubricating oils. As the construction is almost entirely of glass it is possible for the visitor to get an interior knowledge of these processes more intimate than he could obtain by a visit to a real refinery.

In the primary distillation model one may see how the crude oil is heated by means of miniature heat exchangers, and the heated oil is flashed into a fractionating tower for separation into the component parts, such as furnace oil, kerosene, naphtha, and gasoline. The heated incoming crude oil separates into a heavy liquid and vapor. The liquid runs to the bottom of the tower and is drawn off for further processing. The hot vapors circulate among the trays of the tower and, condensing, are run off at different levels. Only the lightest fraction of the ascending vapors remains when the top of the tower is reached, but this is the gasoline.

The catalytic cracking process begins with the oil left over from the primary distillation and which remained at the bottom of the tower. This is heated in a pipe still before it is passed through the catalyst. This cracking process is still a great mystery to the average man and the manner in which it is treated in this model enhances the magical element without in any way diminishing the significance of the actual process. From the cracking cases the vapors travel to another fractionating tower, where the lighter ascend to the higher levels. These, when condensed, give catalytically made gasoline, the better motor fuel.

The mercury vapor process for the manufacture of lubricating oil includes a model of a

mercury boiler (with the flow of mercury shown in transparent tubes), vaporizer and vacuum fractionating tower, a multi-stage injector for vacuum, a water cooled gas oil condenser, coolers, and receiving tanks.

The whole model demonstrates so perfectly the processes involved in the refining of petroleum that it is a source of constant interest to all kinds of museum visitors. Oil men from Russia and Venezuela have been received for special demonstrations. Men of the United States Army Engineers have been instructed by its aid, and a multitude of school children have become familiar with this important industry because it makes a technical subject easily apprehended.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 18, 1950

ACTIVE FAMILY

L. Adrian Doe	Walter B. Kunz	John W. Patton
Richard Williamson Ellis	Edwin Mendelssohn, M.D.	Lew Trenner, Ph.D.

ACTIVE

M. L. Augenblick	George Goodrich	E. Everett Meschter
Thomas W. Averell	William M. Hawkes, Jr.	Alan H. Miller
John A. Bellis, M.D.	Howard Helland	George Patsig
Ralph H. Beter	Daniel S. Hendry	Carl F. Rumpp
Rufus Hayes Cox	W. A. P. Henninger	E. Newlin Sangston
Daniel B. Curll	Frederick R. Jackson	Robert S. Sigel
H. Gilroy Damon	J. R. Keach	James S. Strong, Ph.D.
Robert L. Davis	C. E. Maris, Jr.	William N. Tanner, Jr.
Alfred H. Eckert	Robert C. Marshall	J. Edgar Thompson
Milton L. Godfrey		Melvin C. Webb

ACTIVE NON-RESIDENT

Charles N. Debes	Gerard Erismann	Ernest M. Gilbert

NECROLOGY

E. Paul du Pont '37	Walter T. Grosscup '36	Jordan Mathews '38
R. Max Goepp, M.D. '44	Mrs. Sidney E. Martin '36	William Schlipf '35

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

T. M. CHARLTON: Some Notes on the Analysis of Redundant Systems by Means of the Law of Conservation of Energy.

PARRY MOON AND DOMINA EBERLE SPENCER: Simplified Interflection Calculations.

LUDWIK SILBERSTEIN: Paucity of Quanta Rendering a Silver Halide Grain Developable.

ERIC T. B. GROSS AND LEONARD RABINS: Transient Analysis of Three-Phase Power Systems.

I. MALKIN: Stress Analysis of a Hemispherical Pressure Vessel Head Under Concentric Line Loads.

LEONARD EISENBUD: On the Theory of Angular Phenomena.

PHILIP F. ORDUNG AND HERBERT L. KRAUSS: Frequency-Coordinate Vector Diagram.

The January 1951 issue will be a special number, commemorating the 125th anniversary of the JOURNAL. In addition to the history of the JOURNAL, there will be fifteen papers by outstanding authorities on the subject of *probable future developments in various branches of science and technology*. The complete list of authors will appear in the December JOURNAL, together with the titles of their papers. If extra copies of this important issue are wanted, write to the Assistant Editor stating the number you will want. The cost of each issue will be one dollar, payable either in advance or on delivery.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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BOOK REVIEWS

HANDBOOK OF EXPERIMENTAL STRESS ANALYSIS, M. Hetenyi, editor. 1077 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $15.00.

Members of the Society for Experimental Stress Analysis who have been awaiting the publication of the *Handbook of Experimental Stress Analysis* for several years will not be disappointed, for here is a book containing, or referencing, all the latest information on the various devices of experimental stress analysis, handsomely bound and liberally filled with carefully drawn illustrations and fine photographs. In general, Dr. Hetenyi—appointed by the SESA, who initiated the project, as editor in chief—is to be complimented on so satisfactorily executing his task. It is a noteworthy accomplishment.

Encompassing so broad a field as that of the experimental investigation of mechanical strengths, it was impossible for one individual to write authoritatively on all the aspects required for a comprehensive treatment of the subject. So, recognized leaders in the particular branches of experimental stress analysis prepared, at the invitation of the editor, the various eighteen chapters and three appendices. Such outstanding students of engineering as C. O. Dohenwend, J. H. Goodier, O. J. Horger, R. D. Mindlen, R. E. Peterson, C. R. Soderberg, S. P. Timoshenko and J. B. Wilbur have contributed articles. The book does not lack for competent authorities.

Commencing with the natural introduction to experimental stress analysis, a chapter on the mechanical properties of materials, the *Handbook* proceeds to chapters on testing machines, mechanical gages and extensometers, optical methods of strain measurement, electrical-resistance gages and circuit theory, electric-inductance gages, electric-capacitance gages, motion measurements, strain rosettes, working stresses, residual stresses, methods of crack detection, interpretation of service fractures, brittle models and brittle coatings, structural model analysis, analogies, photoelasticity (both two and three dimensional), and X-ray analysis. The inclusion of appendices on fundamentals of the theory of elasticity, dimensional analysis, and the precision of measurements is exceptionally timely.

While all the subjects are uncommonly and uniformly well presented, being complete with history, theory, construction, application and limitation, depending upon the subject, and are, in themselves, extremely interesting and informative, it is questioned whether all belong in a handbook on experimental stress analysis. A handbook generally connotates a book which has the ready answers to particular problems. Understandably, it is impossible in many cases to present ready answers in such a complex field as the investigation of the mechanical strength of materials. Even so, it hardly seems in order to include chapters on working stresses; methods of crack detection; interpretation of service fractures; the hundred pages or more, exclusive of the rightly included measurement of residual stresses, essentially devoted to the recitation of residual stresses in tubes, cylinders, rails, ingots, gears, crankshafts under various heat treatments; and a large part of the mathematical exposition of analogies.

Containing, as it does, all existing information on experimental stress analysis, the *Handbook* is an invaluable reference book in any field of engineering concerned with the strength of structures. The nearly nine hundred references are presented in most cases with a brief note as to their contents, so that one knows beforehand what the reference contains—a very useful contribution.

E. W. HAMMER, JR.

ELEMENTARY PILE THEORY, by Harry Soodak and Edward C. Campbell. 73 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $2.50.

Three years ago there was presented to a group of scientists and engineers at the Oak Ridge National Laboratory a series of lectures of various phases of atomic energy production and

control. Among these lectures were those given by the senior author, and concerned with the chain reacting pile in which fast neutrons are produced by fission. Declassified versions of the lectures were collected and form the basis for this book. In it there is investigated in detail what happens to the fast neutrons given off in the act of fission, how they move about in space, and how they are slowed down in matter.

The first of three major sections develops the ideas of scattering cross-section and the slowing down of neutrons by reason of collisions with moderator and absorber materials. Attention is then directed to the problem of the space distribution of neutrons and elementary diffusion theory, allowing the computation of the neutron leakage from a chain reacting pile or its critical size. The final major section considers a chain reacting pile operating at a constant level of neutron density. Here there must be a balance between production of neutrons and leakage and absorption. The basic mathematical equations for the pile are given and are solved for the critical pile dimensions for simple pile shapes. The book closes with a discussion of pile reflectors, the use of which can achieve a considerable saving in the initial charge of fissionable material for a pile.

The style of writing is very much that of an informal lecture. Equations and their component parts are explained so that the reader may learn the physical as well as the mathematical reasoning behind them. The book is recommended for engineers and others who would like to have a brief resume of the fundamental ideas of pile theory.

S. CHARP

GERMAN SCIENCE READER, AN ANALYTICAL APPROACH TO TRANSLATION PROBLEMS, by George Radimersky. 245 pages, diagrams, 14 × 21 cm. New York, Ronald Press Company, 1950. Price, $3.00.

Intended for second year students, this textbook presents the author's suggestions for securing facility in the translation of technical German. It is divided into two parts, the first of which discusses certain problems of translation, with illustrative readings, while the second part comprises general readings incorporating several of the problems in one essay.

In his discussion of problems, the author has emphasized certain features of German sentence structure, and has built his teaching method on these, employing the use of diagrams. The manner of stressing these points, which are not limited to scientific German, may convey the erroneous impression that only the budding scientist need master them.

Further, the author has prepared his German text by freely adapting and translating from recent American publications. This is perhaps desirable for Part I where it permits him to illustrate adequately the points treated. However, some potential users of the book may feel that selections from recent German publications would have been preferable in Part II, as presenting actual examples of such writing and also offering a wider variety of styles.

Some typographical errors have been noted such as "Ausgebe" for "Ausgabe" and "Zworkin" for "Zworykin." A useful feature is a glossary of expressions that occur frequently in technical German.

The text will merit consideration by any instructor offering a course in scientific German.

G. E. PETTENGILL

HEAT INSULATION, by Gordon B. Wilkes. 224 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $4.00.

Heat Insulation is in the nature of a handbook, being a short, concise presentation of the more important aspects of heat insulation. It is unusually well written and, as a consequence, the subject matter is particularly clear and easily understood.

Introducing the subject in an interesting manner by describing the purposes of heat insulation and citing several striking examples, the author proceeds to present the fundamental formulas of heat transfer for conduction, radiation and convection. Such formulas include those for conduction through cylindrical, spherical and rectangular surfaces; for radiation from parallel, convex and concave surfaces; and for convection from horizontal cylinders, vertical and horizontal surfaces. Professor Wilkes, with an extensive background in laboratory tech-

nique acquired in the Heat Measurement Laboratory at M. I. T., then describes with authority various methods of determining heat transfer coefficients, several appearing in print for the first time. In chapter four the several factors affecting the coefficient of thermal conductivity are presented. Very practical information is contained in chapters on types of insulating materials, reflective insulation, specific heat of insulating materials, moisture in insulation, and the economics of insulation. Particularly useful, however, is the appendix in which are tables for coefficients of thermal conductivity, specific heat and thermal diffusivity, emissivity and reflectivity of various substances, coefficients of heat transmittance for building walls (including calculated U values), and water-vapor permeability of various materials. The many charts, graphs and fine illustrations contribute to the general usefulness of the book.

Summarizing, *Heat Insulation* provides the engineer and architect with a large amount of practical information on heat flow, heretofore scattered in many sources.

E. W. HAMMER, JR.

ENGINEERING SUPERSONIC AERODYNAMICS, by E. Arthur Bonney. 264 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.00.

The relatively new and rapidly expanding field of supersonic aerodynamics has created many gaps in the necessary background knowledge of the practicing aerodynamicist and aeronautical design engineer alike. The author states that this book is intended to outline to every subsonically educated aeronautical engineer in this field basic information for his background knowledge in supersonics. In this respect, this fine book is a welcome addition to the library of the aeronautical sciences.

In order to give a newly-indoctrinated reader a clear understanding of the subject without the usual complex and too often repeated theoretical derivations, concepts and theories, the author presents the material with an entirely engineering approach, and outlines, with a minimum number of warranted derivations, the practical applied material and the necessary essentials for designing and predicting the performance of supersonic aircraft. The results of the latest applied theoretical work of many authors are presented and these results should serve as useful guides for obtaining aerodynamic data on past and present supersonic design configurations and should also serve as basic tools for enabling the engineer to obtain a "feel" of many other, yet unexplored, wing and body designs.

The material presented rightly avoids concentration on the mathematical details of any specialized branch of this field since it was not intended as a reference text for specialists. Since, in general, the usual subsonic force, stability and performance equations are applicable also to the supersonic field, their derivation and use were not repeated. By presenting excellent physical interpretations and explanations of supersonic flow phenomena, and by deliberately avoiding all the mathematical derivations that can easily be found in many classical texts, Mr. Bonney has avoided the usual "padding" and has therefore provided the newly-educated reader with an excellent continuity of the physical picture of supersonic flow.

The book is divided into seven major chapters. Chapter One contains a concise introduction to the subject of compression and expansion waves. Chapter Two deals with the discussion of one-dimensional flow and is followed by a presentation of two-dimensional characteristics of supersonic flow in Chapter Three. Chapter Four contains the application of the previous material to determine the characteristics of two-dimensional airfoils. Also included at the end of this chapter is a physical discussion of the effects of finite aspect ratio on three-dimensional wing characteristics and the effects of flaps, section, planform and sweepback. Chapter Five covers bodies of revolution, and includes the Taylor and Maccoll conical shock wave solutions along with the linearized theory solution of von Karman, Tsien and Ferrari. Chapter Six introduces the reader to supersonic wind tunnel types, test techniques and procedures, and Chapter Seven discusses the practical physical aspects of composite design and performance including wing-body and wing-tail interference, use of experimental data, stability and control, rockets, ram jets, turbo jets and performance conditions.

SAMUEL M. BERKOWITZ

PHYSICAL CHEMISTRY FOR PREMEDICAL STUDENTS, by John Page Amsden. Second edition, 317 pages, diagrams, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.25.

The material presented in this text can be covered in one semester but it is a thorough and adequate presentation of the elementary principles of physical chemistry and those phases of the subject most valuable to a medical student. The two major changes over the first edition published in 1945, are in the simplified discussion of protolytic equilibrium and the inclusion of a chapter on thermodynamics. This chapter is especially useful in understanding chemical equilibrium and electrode and cell potential. The subject matter in the entire text is very clearly written and the mathematics is kept as simple as possible. In most cases the derivations of the equations used are included. In a few instances these derivations have required the use of calculus, but otherwise the mathematical demands are limited to an ability to solve quadratic equations and to use logarithms. An excellent choice of problems is found at the end of each chapter. Also included in this text is a complete and up to date list of visual materials relevant to the subject matter of each chapter.

A. BUCHY

PRIMARY BATTERIES, by George Wood Vinal. Third edition, 336 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

The title of this book is completely descriptive of its contents, for here is presented a specialized treatment of primary batteries. A primary battery is defined as a source of electric energy which results from electro-chemical action, where the reaction is utilized without regard to reversibility. One or both of the electrodes are consumed as a result of the discharge, and when the battery is in its full state of exhaustion it cannot be restored to its initial condition by passing through it a reversing current.

The author presents a comprehensive picture of his subject. The first chapter is devoted to an historical review of the development of primary batteries from the Volta pile of 1800 to the dry cell industry in 1900. Detailed descriptions and sketches from original papers show the construction of the early cells. This chapter is followed by an elementary discussion of battery theory including the relations of chemical energy to electrical energy and the chemical reactions which transpire in dry cells.

The next three chapters describe in detail dry cells and their operation. Production and manufacturing methods are discussed with attention directed toward the physical properties of raw materials. Comprehensive sections on the subjects of standard cells and standards of emf follow. Here again attention is directed to the important problems of the preparation of materials and the assembly of cells. The effects of temperature and impurities are discussed. The remaining chapters of the book describe new types of batteries, including: air depolarizing batteries; copper oxide-caustic soda batteries; silver oxide and chloride-lead cells; mercuric oxide cells; and fuzed electrolytic cells which are actuated by the application of heat.

This authoritative book will have wide use as a reference work. It should be of particular interest to those in the battery industry and suppliers of raw materials.

S. CHARP

PRINCIPLES AND PRACTICE OF SPECTROCHEMICAL ANALYSIS, by Norman H. Nachtrieb. 324 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.50.

Principles and Practice of Spectrochemical Analysis by Norman H. Nachtrieb is divided into two parts, one dealing with the principles of spectrochemical analysis, the other with the application of spectrographic methods to practical problems.

The first chapter is introductory in character, containing a brief historical sketch and suggested plans for setting up a spectrochemical laboratory. The next two chapters contain discussions of prism and grating spectrographs. Chapter four describes some accessory optical equipment, including lens systems to be used between the source and slit, rotating sectors and microphotometers.

The photographic process and the evaluation of photometric data are then discussed, followed by a brief discussion of direct reading spectrometers such as those using the Geiger-Muller counter and the electron multiplier phototube. Chapter eight deals with the origin of spectra and contains an explanation of the nomenclature and symbols used in atomic spectra.

The second part of the book treats the practical aspects of the subject, introduced by a discussion of the methods for the excitation of spectra. Some commercially available excitation sources are also described. The chapter on the quantitative analysis of metals sets forth the general procedure involved. Methods for the analysis of certain specific materials are included.

There are then three chapters on qualitative analysis, the analysis of refractory materials and the analysis of solutions and soluble salts, respectively.

In the last chapter the importance of chemical separations in spectrochemical analysis is brought out. Chemical separations are frequently necessary, preparatory to spectrochemical determinations since the spectra of some of the trace elements may be altered or masked by the presence of other elements or compounds. Another example cited by the author where preliminary separation is necessary is the analysis of material in which the matrix elements are radioactive. In this case it is important to remove the radioactive elements so that they will not be scattered into the atmosphere during the excitation process. Several methods of separation are discussed.

This book differs from some others preceding it, in that much of the material is presented from the standpoint of the analytical chemist and metallurgist. The description of the instruments involved and the discussion of theory and procedures are adequate but are not emphasized to the point of excluding reference to specific chemical and metallurgical problems.

In the body of the book the chapter on the excitation of spectra is included in the first part, but in the table of contents it is included in the second part on the practice of spectrochemical analysis, where it rightfully belongs. This discrepancy was probably caused by an error on the part of the printer.

This book should prove to be of much value to the practicing spectroscopist.

M. D. EARLE

COLLOID SCIENCE, by James W. McBain. 450 pages, illustrations, 16 × 24 cm. Boston, D. C. Heath & Co., 1950. Distributed by Reinhold Publishing Corp. Price, $8.00.

The field of colloids is presented lucidly and comprehensively in this book. The many practical applications of the physical laws of colloid science suggested in the book point up the increasing significance of the field in industry. Theories, properties, measurements and materials of the science are discussed quite fully. The chapter on emulsions, although connoting nothing new, presents theory and application very capably. The references are voluminous. The chapters on sorption and thin film properties are well organized and authenticated.

There are sections on the optical properties of colloids, brownian movement, ultrafiltration, and lyotropic numbers which go into some theoretical detail. The chapters on rheology are rather scanty for so extensive a subject.

The second half of the book is devoted, to some extent, to a study of the colloidal nature of such common materials as soaps, cellulosic materials, starch, gums, rubber, clays, proteins like silk and wool, and aerosols.

The scope of this book is as broad as the entire field, and with its ready references, is to be highly recommended for anyone interested in the study of colloids.

S. N. MUCHNICK

METHODS IN CLIMATOLOGY, by V. Conrad and L. W. Pollak. 459 pages, illustrations, 16 × 24 cm. Cambridge, Harvard University Press, 1950. Price, $7.50.

Methods in Climatology is essentially a text covering the methods to be used in handling problems in which the data available are of a statistical nature. As such, the book is useful to anyone faced with problems of this type, although the text, of course, refers and directs all of the methods discussed to the specific field of climatology.

Part I of the book covers statistical methods, tests, and criteria in some detail. It also covers curve fitting problems and harmonic analysis. In addition there are chapters in Part I

devoted to the characteristics of climatic elements, spatial comparison of climatic elements, and essential elements of climatography. Part II is devoted entirely to computing devices and to the methods of periodography including some information on autocorrelation methods.

The text is well written, well illustrated, and presents a number of practical examples of the type of problems which the climatologist can expect to encounter.

R. S. GRUBMEYER

THE ROLLING OF METALS, THEORY AND EXPERIMENT, by L. R. Underwood. Volume 1, 344 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1950. Price $6.50.

Written by the chief engineer of the Rolling Mill Department of W. H. A. Robertson & Co., England, *The Rolling of Metals* is a thorough account of the theory, experiment and much of the history of metal rolling practice. In preparing the book, which is the result of a series of papers on the rolling of metals, Dr. Underwood has reviewed all existing material on the subject so that the final product is exceptionally complete. For years the technique of rolling metals was acquired by slow trial and error methods and existed primarily in the minds of the mill foremen. It is only in the past generation that much scientific study has been instigated; all such scientific studies are fully referenced and discussed by Dr. Underwood.

The subjects investigated in *The Rolling of Metals* include the forces acting in the region of contact between the material and the rolls, forward slip, the flow of material in rolling, yield stress and the criteria of yielding, external friction between the rolls and the material, factors influencing rolling load and specific roll pressure, theories of rolling applicable to thin sheet and strip, calculation of specific roll pressure for strip and sheet with strip tension and work-hardening by means of the preceding theories, and roll flattening and the calculation of specific roll pressure with flattening. The bibliography, classified according to the various subjects just listed, plus several others, should be a valuable reference for those concerned with the problems of rolling mills.

Throughout the volume the sundry theories on the many aspects of the rolling of flat metals are presented, usually developed and compared. Where necessary, additional research to fill the gaps in the present theory is suggested and frequently outlined. Frequent examples are given, illustrating the use of the formulas, a not unimportant contribution, so that the book has a very practical, as well as theoretical, value to the rolling mill profession.

E. W. HAMMER, JR.

THE PROPERTIES OF METALLIC MATERIALS AT LOW TEMPERATURE, by P. Litherland Teed. 222 pages, 14 × 23 cm. New York, John Wiley & Sons, Inc., 1950. Price, $3.50.

As indicated in the title, this book is a compilation of data on the properties of metals and alloys at low temperatures. Near the beginning of the book the author mentions the design of aircraft intended to operate at high altitudes as one of the chief fields in which a knowledge of the low temperature properties of metals is important. He also mentions several commercial processes in which temperatures as low as − 200° C. are reached, where the physical properties of metals at these low temperautres must be taken into consideration.

In the second chapter the author discusses the relation between physical properties and temperature in terms of physical laws. He states, however, that in spite of the great advances which have been made, it is impossible to calculate with any degree of reliability the properties of a material at one temperature if these same properties at another temperature are known.

The next seven chapters deal with the mechanical properties of a wide variety of alloys at low temperatures. Among the materials considered are: aluminum, aluminum alloys, ferrous alloys, magnesium alloys, copper, copper alloys, nickel, zinc, tin and lead.

Some of the properties whose temperature dependence is discussed are: yield stress, ultimate stress, reduction of area, fatigue stress, electrical resistance, and impact strength. The changes in the various moduli of elasticity with temperature are also discussed.

The last chapter is written as a conclusion. The book contains a large number of tables and the author is quite thorough in listing the sources from which he obtained his data.

This book should be useful to the mechanical engineer interested in the design of structures and equipment which must withstand low temperatures.

M. D. EARLE

CIRCUITS IN ELECTRICAL ENGINEERING, by Charles R. Vail. 560 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $5.75.

This author has managed to write a good elementary text on electrical circuit fundamentals without once mentioning electronics or vacuum tubes. At this time few electrical writers, even in the electrical power field, are able to refrain from referring to these popular subjects.

Covering a huge amount of territory with brief explanations of microscopic physical phenomena underlying macroscopic electrical effects, the volume assumes nothing about the student's general knowledge or background. All questions of units and the origin of dimensional quantities are carefully tabulated so that the student need have no vague feeling about the correspondence among these quantities.

In general, the treatment of circuits having linear parameters is the rule and only brief reference is made to nonlinear elements. However, the mathematical treatment of nonsinusoidal waves is covered in some detail owing to the fact that the emf wave from a commercial alternator is rarely exactly sinusoidal, even at no load, and the effect of armature reaction in distorting the flux distribution as the machine acquires load can result in a considerable distortion of the emf wave. Examples of rectangular and special wave forms are used to show how they may be studied by both analytical and graphical methods. The care with which the Fischer-Hinnem method is demonstrated deserves special mention. The transient behavior of circuits in which a circuit element or emf has been inserted or removed has been fully covered owing to the close relation of this subject to power system switching.

Since this is a text book for undergraduate instruction purposes, each chapter concludes with a set of problems. As many numerical examples are given showing the details of the solution. The author has no fear of degrading his book by simplification. His teaching is graphical and one gets the impression that nothing has been spared so that clarity may be achieved. Nor has he any fear of presenting "old stuff" if it is something that should be mastered by the student.

The book should be an easy one from which to teach a course, for the many subjects discussed have been carefully sorted and compartmentalized into convenient teaching units. In one respect, such a large number of details are covered that the book bears some resemblance to an encyclopedia of elementary electricity and engineering principles.

C. W. HARGENS

PROCESS HEAT TRANSFER, by Donald Q. Kern. 871 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $8.00.

Text books on engineering subjects should always be written by men with practical as well as theoretical backgrounds; for, generally, such a combination yields a volume of much greater usefulness. Mr. Kern, as director of the Process Engineering Division of the Patterson Foundry & Machine Company and adjunct professor of chemical engineering at Polytechnic Institute of Brooklyn, has such a background, and his book, *Process Heat Transfer*, fortunately reflects his varied experience. Admirably combining theory with practice, Mr. Kern has prepared a text book for undergraduate and graduate use, written in the language of industry, which introduces to the student many of the industrial methods in process heat transfer study. Of course, the theoretical treatment of the common methods of heat transfer as conduction, convection and radiation is not unique, and is, essentially, no different from the presentation in other standard texts, with the noteworthy exception, however, that Mr. Kern has wisely omitted theoretical material of no practical use in industry. But combined in a single volume with descriptions of such process equipment as vaporizers, evaporators, and cooling towers, and various methods for calculating such industrial process equipment as radiant heat furnaces, jacketed vessels, coolers, and exchangers, the book provides the engineer and student with a rather complete concept of heat transfer and its application to the chemical process industries.

Process heat transfer differs from heat transfer, which treats generally of the rates of heat exchange between heat sources and receivers, by dealing specifically with the rates of heat exchange occurring in the heat-transfer equipment of the engineering and chemical processes. Professor Kern introduces his subject with chapters on process heat transfer, conduction, convection, radiation and temperature, in which the theoretical treatment is freely supplemented

with the solution of practical examples. Following this are chapters on the theory and design of double-pipe exchangers and shell-and-tube exchangers and their flow arrangements for increased heat recovery. Calculations for compressors using gases instead of liquids for heating or cooling are presented in the chapter on gases. The determination of the optimum heat exchanger for particular process conditions; the condensation of single and mixed vapors; the phenomenon of evaporization; the types, sizes and design of vaporizers, evaporators, reboilers and cooling towers; and the design of various extended surfaces to increase the surface available for heat transfer are subsequently discussed. The foregoing material presented concerns steady state processes, or those assumed as steady state. In the chapter on batch processes, the most common unsteady state processes encountered in heating and cooling solids are explained. Some of the empirical and semi-theoretical methods for the calculation of the radiant-heat transfer in the design of steam-generating boilers and petroleum furnaces occur in the very useful chapter on furnace calculations. The design of such heat transfer equipment as coils, trombone and atmospheric coolers, and electric-resistance heaters are also treated.

Liberally dispersed throughout the volume are many excellent illustrations of common process equipment and practical examples of the design of such equipment. In addition to the design data in the form of charts and tables included in each of the chapters, is an appendix of some fifty pages containing such practical data as thermal conductivities, specific heats and gravities, ethalpies and latent heats, viscosities, heat transfer coefficients and heat-exchanger and condenser-tube data. Unusually thoughtful and helpful, too, is the inclusion, at the end of each chapter, of a list of the symbols used in each chapter. To anyone who has looked with increasing annoyance through a book for the definition of some obscure symbol, such a feature is a pleasure.

E. W. HAMMER, JR.

THE PRINCIPLES OF SCIENTIFIC RESEARCH, by Paul Freedman. 222 pages, 15 × 22 cm. Washington, Public Affairs Press, 1950. Price, $3.25.

The author of this book has written it primarily for two different groups of people: those young scientists who are contemplating entering the field of research, and those business executives who have not realized what benefits may be reaped from a research program, or those executives who, having established a research program, attempt to run the program on a production line basis. Although the book is addressed mainly to these groups, it is of great interest to anyone currently engaged in scientific research. In the text can be found many of the truths which the scientific worker "knew all the time," but of which he may not have been fully aware or which he may not have integrated into the pattern he follows in the pursuit of scientific truths.

The opening chapter is devoted to "The Nature of Research and its History." Not only in this chapter but throughout the remainder of the book will be found interesting bits of the history of the development of the research method. It may at first surprise the reader to find that the history of scientific research is a history of philosophy, and not a history of great inventions or even of great inventors. The author has done an excellent job here in not allowing these two subjects to become confused.

In ensuing chapters, the author discusses "Research and Society," "Research and Philosophy," and "The Mental Approach." The next two chapters are a very well written exposition on "The Planning of Research," well worth reading and rereading by anyone engaged in research work. The remaining chapters deal with "Organization," "Experimentation," "Accuracy and Economy of Effort," "Minimum Number of Essential Observations," and "Patrons." There are no rock-bound rules or conclusions set forth here, but the author has erected many sign posts pointing in the direction of a more thorough understanding of all phases of a difficult subject: scientific research.

Throughout the text, the author uses interesting practical illustrations, presumably drawn from his experience as a scientist, to clarify and amplify the points under discussion. This technique, coupled with a style of writing which is smooth and free-flowing, makes the book enjoyable as well as educational reading.

R. S. GRUBMEYER

METHUEN'S MONOGRAPHS ON PHYSICAL SUBJECTS. Illustrations, 11 × 17 cm. London, Methuen & Co. Distributed in U. S. by John Wiley & Sons, Inc., New York.

Relativity Physics, by W. H. McCrea. 87 pages. 1950. Price, $1.25.

The Special Theory of Relativity, by Herbert Dingle. 94 pages. 1950. Price, $1.25.

These two volumes are complementary but distinct. The first title furnishes deductions and standard derivations of the results of relativity theory which find most frequent application in physics. These include relativity kinematics, mechanics, optics, electromagnetic theory; atomic physics; and thermodynamics, statistical mechanics, and hydrodynamics. The basic equations are derived and in many cases references are made to applications of this theory by others.

The second volume bases its treatment on the measurement of lengths. Except for very clear descriptions of some famous experiments, including the Fizeau experiment, the Michelson-Morley experiment, and those by Kennedy and Thorndike, and Miller, the subject matter is very much the same as that of the first title.

Both volumes are written as reviews for physicists. They are so recommended.

Electromagnetic Waves, by F. W. G. White. 108 pages. 1950. Price, $1.25.

Wave Guides, by H. R. L. Lamont. 118 pages. 1950. Price, $1.50.

These two monographs present in short, but comprehensive form, the fundamental mathematical theory of the propagation of electromagnetic waves in free space, in dispersive media, in the ionosphere, and radiated from wave guides. Both books are concerned with special solutions of Maxwell's equations, in the first case only for plane waves and in the second case with respect only to wave guide transmission theory and wave guides used as radiators, resonators, and lumped constant circuit elements. While the second book is primarily theoretical, the first book in many instances makes comparisons with experimental results for the cases of the optical properites of metal, the dispersion of X-rays, and the equivalent height of ionizing layers.

Both volumes are recommended for those who wish to have nearby in review form these special solutions of Maxwell's equations.

S. CHARP

BOOK NOTES

THE PHILOSOPHY OF MATHEMATICS, by Edward A. Maziarz. 286 pages, 14 × 22 cm. New York, Philosophical Library, 1950. Price, $4.00.

Conceived originally as a doctoral thesis at the University of Ottawa, the present work aims to meet a present widespread interest in the philosophy of mathematics. An introductory chapter endeavors to clarify the notion of the philosophy of mathematics by placing it in a wider speculative setting; then follows an historical part which considers the question of what mathematics is as viewed by both past and recent thinkers; a third part, doctrinal in nature, discusses the main questions raised by a philosophical analysis of the nature of mathematics.

The work fairly bristles with "superior numbers" and as a result more than one third the textual pages are devoted to notes. An extensive bibliography is also included.

CALCULUS AND ANALYTIC GEOMETRY, by Cecil Thomas Holmes. 416 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.75.

This text is intended to serve as a basic course in calculus and analytic geometry, both for the liberal arts student, who desires an idea of what it is all about, and for the prospective engineer or scientist, who will need the knowledge in his business. At Bowdoin, where the author teaches, the first five chapters are used as a one semester course, serving as a *terminal* course in mathematics for some students. Those who go on in the calculus sequence secure the remainder of their introductory course from the rest of the book. This latter part offers more than the normal course will allow time for, so that the instructor may have freedom of choice in selecting his teaching material to meet the needs of his class. Many problems are included, with answers for some. The author has made an interesting presentation of his material.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

The Inhibition of the Lecithinase of *Clostridium perfringens* by Some Reducing Agents.—LOUIS DESPAIN SMITH AND MARY VIRGINIA GARDNER. Although considerable work has been done on the lecithinase (alpha toxin) of *Clostridium perfringens*, several aspects of its behavior remain somewhat puzzling. For example, MacLennan and Macfarlane (1) have reported that *C. perfringens* lecithinase was strongly adsorbed by living muscle but that muscle removed from an animal within a few minutes after death did not adsorb lecithinase. It seemed to us that the property of muscle which would be most likely to change rapidly enough after death to cause this phenomenon would be the oxidation-reduction potential, which might be expected to drop rather sharply. Consequently, we investigated the effect of some reducing agents on the activity of the lecithinase of *C. perfringens*. The results of this investigation, which are reported here, indicate that lecithinase could be completely, and apparently irreversibly, inactivated by certain reducing agents.

METHODS AND MATERIALS

The lecithinase preparations were similar to those described by Smith and Gardner (2), except that they had been preserved by the addition of two volumes of glycerin. Lecithinase determinations were performed by the lecitho-vitellin test described by van Heyningen (3). Determinations of hemolytic activity were performed according to the method described by Bernheimer (4), except that mouse erythrocytes were used instead of rabbit erythrocytes because of the greater resistance of the former to theta toxin. Sodium chloride solution (0.15 *M*) containing 0.005 *M* calcium acetate was used in place of phosphate buffer. It should be pointed out that the erythrocytes from the strain of mice that we used were somewhat susceptible to theta toxin hemolysis. Consequently, a small amount of cholesterol was added to the lecithinase preparations used for hemolytic and toxicity tests. Adequate control tubes were included in the experiments on hemolysis to demonstrate that theta toxin was not responsible for the hemolysis observed.

The amount of intravascular hemolysis was determined by opening the chest cavities of the mice immediately after death, removing a small amount—usually about 0.05 ml.—of blood from the heart, suspending this blood in a measured quantity of oxalated saline solution, and centrifuging. The supernatant fluid was removed, diluted to 20 ml. with 0.15 *M* NaCl solution, and the absorption determined in a Klett colorimeter, using the No. 42 filter. The sedimented cells were lysed

with the aid of a drop of ether, diluted to the same volume as was the supernatant fluid, and the absorption similarly determined. From these two readings, the hemolysis existing in the heart blood was determined. Control animals which were killed by ether, chloroform, or trauma exhibited no detectable intravascular hemolysis.

EXPERIMENTAL

Stock lecithinase solution was diluted in borate buffer pH 6.9 and freshly neutralized solutions of cysteine hydrochloride were added to give 12 egg units (E.U.) of lecithinase per ml. and the desired concentrations of cysteine. After standing 15 min. at room temperature, the mixtures were titrated for lecithinase activity. The results, shown in

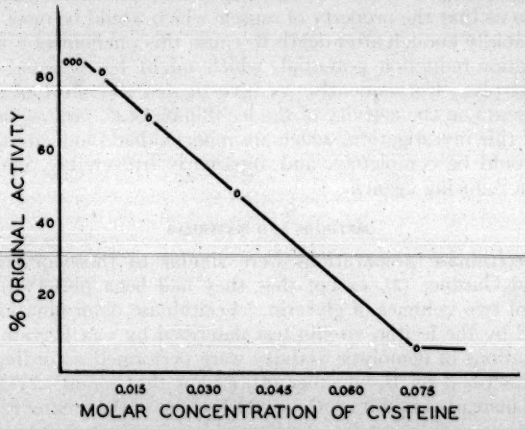

FIG. 1. Inhibition of lecithinase of *C. perfringens* by cysteine.

Fig. 1, indicate that the lecithinase activity was definitely decreased by cysteine and that, within certain limits, the decrease of lecithinase activity was proportional to the concentration of cysteine. This experiment was repeated with glutathione as reducing agent; the mixtures were allowed to stand for various periods of time before lecithinase activity was determined, with the results shown in Fig. 2. The apparent linear relationship between loss of lecithinase activity and concentration of cysteine or glutathione should not be interpreted as implying a mol-for-mol reaction between the lecithinase and the reducing agent, for it is probable that the impurities also reacted with the reducing agent.

It was found when other preparations of lecithinase were used that the concentration of cysteine or glutathione required to bring about the same per cent inactivation varied from one preparation to another. This

state of affairs would be expected if the assumption made in the foregoing paragraph were true, since varying amounts of impurities would be expected to be present in different preparations, even though the active lecithinase content were the same.

It seemed of interest to determine whether cysteine would inhibit lecithinase activity in the presence of substrate. Figure 3 shows the results of such an experiment, in which sufficient freshly neutralized cysteine was added after the enzyme had started to act on its substrate to bring the concentration of cysteine to 0.033 *M*, a level which was expected to reduce lecithinase activity to about one-half the original value (see Fig. 1). The enzyme was in concentration of 2.4 E.U. per ml. The turbidity values were calculated to allow for the small drop in

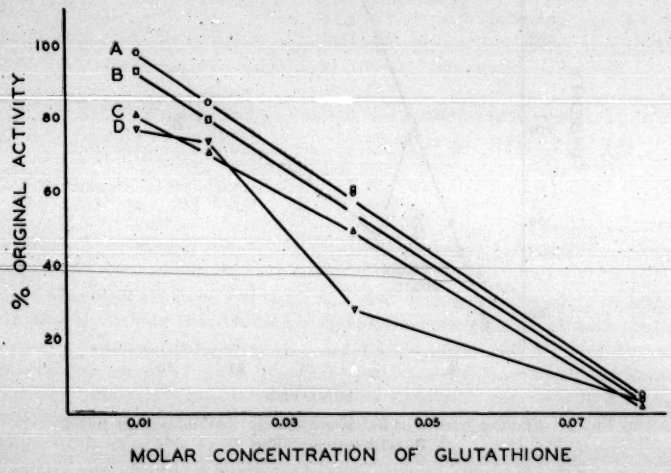

FIG. 2. Inhibition of lecithinase of *C. perfringens* by glutathione. Length of time of exposure of lecithinase to glutathione before testing: A = 5 min.; B = 15 min.; C = 30 min.; D = 60 min.

turbidity caused by the addition of a volume of buffer equal to the volume of cysteine solution. The initial drop in turbidity shown in Fig. 3 was found to be the result of the action of cysteine on the end-products of the enzyme reaction, and not of a dilution effect. The decrease in lecithinase activity is indicated by the difference in the slopes of the two curves, a decrease in activity of about fifty per cent. Apparently, the presence of substrate had little effect on the inactivation of the enzyme by cysteine.

Since the hemolytic and toxic activities of *C. perfringens* culture filtrates are probably referrable to the ability of the lecithinase to dis-

rupt lipid-protein complexes (5) the inhibition of hemolytic and toxic activity was investigated. The hemolytic activity of solutions of lecithinase was found to be reduced by cysteine or glutathione. However,

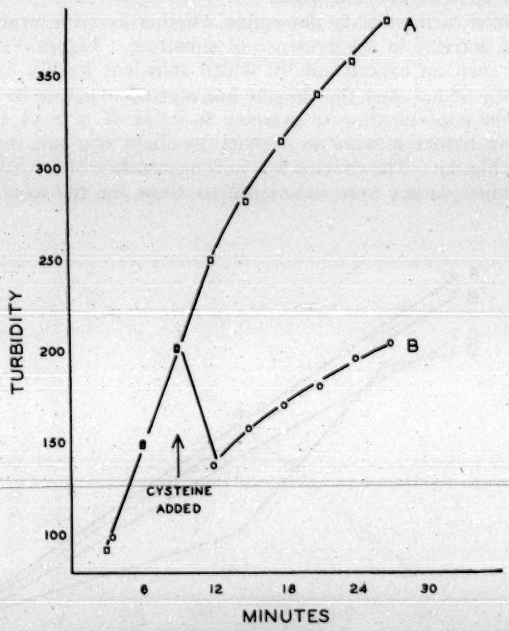

FIG. 3. Effect of cysteine on lecithinase activity. A—no cysteine added. B—with cysteine added.

the rate of decrease in hemolytic activity was not directly proportional to the rate of decrease in lecithinase activity, as the data presented in Table I indicate. In this experiment, the lecithinase was used in con-

TABLE I.—*Inhibition of Hemolytic and Lecithinase Activity by Glutathione.*

Concentration of Glutathione	% Original Activity	
	Hemolytic	Lecithinase
0.092 *M*	0	0
0.046 *M*	18.8	0
0.023 *M*	50.0	5.0
0.0115 *M*	83.0	39.0

centration of 9.4 E.U. per ml., and was allowed to stand together with the glutathione for 1 hr. at room temperature before being diluted for immediate testing. The preparation of lecithinase used in this experi-

ment contained but very little theta toxin. The preparation used in the other experiments showed appreciable theta toxin activity after being treated with cysteine or glutathione.

The effect of glutathione on the toxicity of a lecithinase preparation was determined by diluting the stock lecithinase in freshly neutralized glutathione solution to give a final concentration of 0.1 *M* glutathione and a toxin concentration of 20 LD_{50} per ml. (13.3 E.U.). Cholesterol (about 0.00013 *M*) was added to inhibit theta toxin activity. This mixture was allowed to stand at room temperature for definite periods of time, and was then injected into mice intraperitoneally, in 0.5 ml. doses, with 5 mice to a group. The toxic activity of the mixture slowly declined, as the data in Table II indicate. Comparison of these data with those in Fig. 2 indicates that the toxicity declined more slowly than did the lecithinase activity. When the mixture of lecithinase and 0.1 *M* glutathione was allowed to stand overnight in the cold, all toxic activity as well as hemolytic and lecithinase activity was found to be lost.

TABLE II.—*Inhibition of Toxic Activity of Lecithinase by Glutathione.*

Length of Time Mixture was Allowed to Stand before Injection	Mean Time between Injection and Death ±P.E. (in minutes)	Mean Value for Intravascular Hemolysis ±P.E. (in %)
Control (no glutathione)	53± 4.9	26.8±0.8
1 to 3 min.	96± 9.6	16.3±4.4
30 min.	110±15.1	19.7±1.6
60 min.	234± 6.8	14.8±1.5

It also appears from the data in Table II that the amount of intravascular hemolysis was somewhat reduced by the treatment with glutathione. When rabbits were used instead of mice, and the lecithinase-glutathione mixtures were injected intravenously instead of intraperitoneally, essentially similar results were obtained, *viz.*, the lecithinase activity was more greatly affected by short exposure to 0.1 *M* glutathione than was the hemolytic or toxic activity, and sufficiently long exposure abolished toxic and hemolytic activity also. In all the experiments, both with rabbits and with mice, it was evident that the effect of the reducing agent on toxic and hemolytic activity was approximately the same; the effect on lecithin-splitting activity was always much greater.

In an effort to determine whether partially inactivated lecithinase was appreciably changed so far as serological characteristics were concerned, a solution containing 40 mouse LD_{50} doses of toxin per ml. was mixed with an equal volume of 0.1 *M* glutathione and allowed to stand 15 min. at room temperature. It was then divided into two portions; to one portion sufficient antitoxin to neutralize the lecithinase was added, to the other 0.15 *M* NaCl solution. Volumes were adjusted so that the final concentration of lecithinase was 20 LD_{50} per ml., disregarding the

inactivation. A dose of 0.5 ml. of the lecithinase-glutathione-antitoxin mixture was injected intraperitoneally into each of 10 mice. A similar dose of the lecithinase-glutathione mixture was injected into each of another group of 10 mice. All of the mice in the first group survived; all of the mice in the second group died within 24 hr. Apparently, partial inactivation of the lecithinase did not change the serological specificity of the toxin which still remained active.

Some reducing agents other than cysteine and glutathione were also effective in reducing lecithinase activity. Thioglycolic acid, sodium bisulfite, and hydrogen sulfide were active in this regard. Ascorbic acid, however, affected the lecithinase activity but slightly, even in 0.5 *M* concentration. It was not possible to reduce lecithinase to an inactive form by means of platinum black and hydrogen at 1 atmosphere pressure.

Numerous attempts were made to reverse the inactivating effect of cysteine, glutathione, and thioglycolic acid on lecithinase by treatment of partially inactivated solutions with hydrogen peroxide, oxygen, oxidized glutathione, by exposure to the atmosphere in the cold, by heating at 100° C. for 5 min., or by dilution. In each case, the lecithinase was dissolved in neutral solution. No unquestionable and repeatable reversal of inactivation was noted. This lack of reversibility and the inability of ascorbic acid or hydrogen and platinum black to inactivate lecithinase indicate that the situation is not simply one of oxidation-reduction potential. Rather, it would appear that the enzymatically active portion of the lecithinase molecule may contain one or more disulfide groups whose integrity is essential to enzymic activity, in a manner analogous to the disulfide groups found to be necessary for the biological action of insulin by du Vigneaud *et al.* (6).

The lack of proportionality between the action of partially inactivated lecithinase on red blood cells and lecitho-vitellin is a puzzling phenomenon. However, it should be pointed out that Miles and Miles (7) have reported that the lecithin-splitting ability of filtrates of various strains of *Clostridium bifermentans* was not parallel to the hemolytic or toxic activity. These latter properties, however, were proportional, just as has been found for *C. perfringens* lecithinase partially inactivated by reducing agents.

In view of the fact that the lecithins are a group of compounds, rather than a single one, these findings raise the question as to whether different lecithins are involved in the action of lecithinase on red blood cells and on lecitho-vitellin, or whether a mixture of lecithinases is involved. Until more knowledge is accumulated regarding the fatty acid composition of lecithins from various sources and the action on them of various lecithinases, this question cannot be answered satisfactorily.

SUMMARY

1. The lecithinase of *Clostridium perfringens* was inactivated by cysteine, glutathione, thioglycolic acid, hydrogen sulfide, and sodium bisulfite. This inactivation was not found to be reversible.

2. It was not inactivated by ascorbic acid or by hydrogen and platinum black.

3. With low concentrations of glutathione, the action of lecithinase on lecitho-vitellin was inhibited to a greater degree than was the action on red blood cells or the toxic action.

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CURRENT TOPICS

The Colloquium on Mesospheric Physics.—A Colloquium on Mesospheric Physics[1] was held at Cambridge, Mass., on 31 July–1 August 1950, under the auspices of the Geophysical Research Directorate, AF Cambridge Research Laboratories. This two-day session was highlighted by the presence of an outstanding group of international authorities in the field of mesospheric problems, including D. R. Bates, University College, London, England; S. Chapman, Oxford University, Oxford, England; W. Dieminger, Institüt für Ionosphärenforschung, Lindau, Germany; L. Harang, Norwegien Defense Research Establishment, Kjeller, Norway; L. G. H. Huxley, Adelaide University, Adelaide, South Australia; D. F. Martyn, Australian Council for Scientific and Industrial Research, Canberra, Australia; S. K. Mitra, University College of Science and Technology, Calcutta, India; J. Sayers, The University, Birmingham, England; K. Weekes, Cambridge University, Cambridge, England; and R. v.d. R. Woolley, Commonwealth Observatory, Canberra, Australia. In addition, the Colloquium was fortunate in having many distinguished local attendees, including H. Lettau, M. O'Day, R. Penndorf and W. Pfister of the Geophysical Research Directorate; D. H. Menzel, H. R. Mimno, and F. Whipple of Harvard University; S. C. Brown, Massachusetts Institute of Technology; W. H. Bostick, Tufts College, Medford, Mass.; D. E. Kerr, Johns Hopkins University, Baltimore, Maryland; A. H. Shapley, Central Radio Propagation Laboratory, Washington, D. C.; and several others.

This Colloquium was initiated by the AF Cambridge Research Laboratories to stimulate interest and encourage activity in a field which has tremendous research opportunities. In the small group indicated above, a free flow of thoughts definitely delimited the small extent of current knowledge in the field of Mesospheric Physics and indicated the large area where active research is vitally necessary.

The Colloquium was opened by the topic "Mesospheric Properties" under the chairmanship of F. L. Whipple. The enormous difficulties involved in the obtainment of samples of the upper atmosphere for a determination of constitution were discussed. A novel method of employing a balloon- or rocket-borne liquid hydrogen trap which could be opened at definite levels and thereby provide frozen air samples from given altitudes was mentioned. The complicating effects of absorption and adsorption by the container in the usual sampling procedure, or of the heat of association (in the case of atomic oxygen) upon the frozen sampling methods were brought out. The possibility of employing a vehicle-borne mass spectrograph for determination of composition was also examined. While it was thought that water vapor is largely dissociated at about 75 km. and above, detailed theoretical studies on the composition supplemented by experimental results are desirable. The use of interference filters in the infrared to study water vapor was proposed.

Meteor observations currently indicate higher atmospheric number densities in the region 50–90 km. than those determined directly by means of rockets. A seasonal and latitudinal variation in the number density of the atmosphere at higher altitudes probably exists. With respect to noctilucent clouds, if water vapor is dissociated at the level concerned, meteoric flotsam or cosmic dust

[1] The Mesosphere is considered to be that region from the base of the ionosphere to the critical level of escape; thus, the mesosphere lies in the altitude range of about 70–1000 km.

may be the principal formative agent. However, it was concluded that the effects of contaminants may markedly affect dissociation; also, small dust particles at 100 km. would fall rapidly, although the rate of fall at altitudes of 70–80 km. would be somewhat smaller. Possible temperature gradients above 100 km., and the possible connection between temperature increase with height (in the E and F_1 region) and the sunspot number was mentioned. The difficulties encountered by the use of scale height to determine temperature in the ionospheric layers was emphasized. These difficulties arise because of changes in the shape of the layer, neglect of the magnetic field, etc.

The role of the photosphere and chromosphere in affecting the ionosphere and ozonosphere was discussed. Problems in deducing upper atmospheric characteristics from meteoric data were outlined.

The afternoon session was devoted to "Ionospheric Mechanisms and Reactions," under the chairmanship of J. Sayers. The most definite evidence on the constituents of the ionospheric layers comes from the "dawn flash" of the first negative system of nitrogen. Estimates of the number density of ionized molecular nitrogen N_2^+ then are about $10/cm^3$. On the basis that molecular nitrogen is not involved in the formation of the ionized layers, ionized atomic oxygen must form both the F_1 and F_2 regions. E layer ionization may be produced mainly by pre-ionization of molecular oxygen. An alternative theory is possible: that atomic oxygen forms the E layer, in which case there is great difficulty in deducing the ionized constituents forming the F regions. Although ionized atomic nitrogen is a possibility, this constituent may not exist in sufficient number density. There is no clear-cut evidence to distinguish between the rival theories. The night sky gives no evidence of the existence of atomic nitrogen. However, auroral data indicate that molecular nitrogen exists to extreme heights. The question on the relative proportions of atomic to molecular nitrogen at different mesosphere levels was not resolved.

The persistence of long enduring meteor trains may possibly be attributed to the more than the normal amount of sodium in the meteor concerned.

Evidence points to electric current systems at an altitude somewhat below the E layer as the principal factor in transient variations of the magnetic field observed at the earth's surface. On this basis, about 10^4 negative ions per electron is taken below the E layer, too rapid a disappearance of charges ensues. No method of resolving this discrepancy is apparent at present. The necessity of increased investigations of ionospheric tides was mentioned.

Recent laboratory measurements of the recombination coefficient by microwave techniques undertaken at relatively high pressures and leaving some doubt as to the mechanisms involved, do not seem to be applicable to the ionosphere. The semi-diurnal variation in the lunar tide of about 2–5 km. in the F_2 region was considered well substantiated by experimental evidence. The possibility of a pressure dependent recombination coefficient and its effect, together with that of tides, in changing the electron density in the F_2 layer was outlined.

The third session "Solar Terrestrial Relationships" under the chairmanship of R. v.d. R. Woolley opened on 1 August at the Harvard Astronomical Observatory. As the temperature of the photosphere (at a wavelength of about 1000 angstrom units) is about 4830° K., insufficient quanta from the photosphere are present to produce the terrestrial ionosphere. However, as the coronal temperature attains a value of 10^6° K., many high energy quanta are available hwich presumably could cause the ionospheric layers. Thus, a thorough and

detailed examination of the ionosphere should provide information regarding the upper energy limit of quanta emitted by the sun. However, it is currently not possible to deduce solar activity from observed ionospheric changes; on the contrary it is necessary to examine in great detail the solar spectrum in order to deduce ionospheric and mesospheric mechanisms. A most desirable if not absolutely necessary rocket experiment is that of determining the solar emission spectrum during quiet and disturbed periods in the wavelength region from about 1 to 2000 angstrom units. Solar emission in the Lyman alpha bands and the magnesium pairs and the possibility of photoelectric measurements in the ultraviolet and X-ray regions were mentioned. The expected opacity of the ionosphere was given as: F_2 region transparent to about 13.5 eV but possibly not transparent to 15.5 eV. The cross section for absorption is uncertain in the F_1 region, so that quanta of 13.5 eV may penetrate to the E region. Relationships between flares and ionospheric activity were discussed. While atmospheric absorption of both the ultraviolet and microwave emissions of the sun may be studied, the much more profitable region for investigation is the former. Undetectable ionizations (in the mesosphere) may occur which might account for a heating of the ionospheric layers.

It was suggested that spicule activity near the solar poles may be of extreme importance in both radiational and corpuscular activity. A preliminary study indicates that the velocity and number of spicules increase with sunspot activity. No evidence exists regarding extensive emissions of corpuscles from solar flares or the sun itself since most ejected material apparently returns to the solar surface. The possibility that the penetration power of particles causing the aurora arises from magnetic energy transformed into kinetic energy does not fit geomagnetic evidence.

The final session of the Colloquium was entitled "Mesospheric Dynamics" and was held under the chairmanship of R. Penndorf. Research on vertical motions of visual meteor trails in the altitude range 70–100 km. is currently being attempted. It is not possible to distinguish clearly between vertical and horizontal motions. Meteor observations to date indicate average speeds of about 48–57 m/s. Over North America, the drift seems to be predominantly from the South with components from the East, whereas in Europe mainly from the East. The determination of winds and motions by the release from rockets of thin aluminum leaf or possibly sodium vapor under certain conditions was deemed much superior to the use of smoke which diffuses very rapidly.

Indications of motion near an altitude of 100 km. were presented utilizing radio amateur observations on sporadic E reflections. Movements of the center of mass of the sporadic E area are obtained. Initial results for June 1949 reveal a preferred direction of motion from the East, although winds from all directions have been observed. The movement of the center of mass of the area averages about 65 m/s. Sporadic E motions are characterized by a sudden onset, or appearance of the reflection area, followed by somewhat rapid movement, then dissipation. The major portion of sporadic E was reported during the interval May–August.

Wind motions below the altitude of the E region have been obtained in England. A complicated quotidian variation of the wind vector throughout the day has been found. No lunar variation is thus far in evidence. Wind velocities range from 80 m/s to lower values but the trend in motion is maintained. Over Germany, apparent motions of ion clouds are 15–200 m/s in summer (during the day), with directions variable. Different wind velocities

in the E and F layers were reported. The possibility was mentioned that these apparent motions represent not actual wind motions, but the travel of a disturbance or atmospheric condition. A study of the movement of ionospheric disturbances in the F_2 region over Australia indicated something like a "monsoon effect" with winds from the NW during winter and SW during summer. No reliable evidence of a diurnal variation was obtained. Movements of "Leuchstreifen" over Germany at a height of 120 km. revealed speeds of 50–90 m/s. Auroral clouds occasionally seen in Norway indicated speeds of 50–100 m/s.

With respect to atmospheric diffusion, a most difficult problem exists. A main problem in diffusion is the temperature changes which arise from solar radiations. A generalization to three dimensional global diffusion is very complex. During the Krakatoa eruption, two-dimensional diffusion occurred at a given level followed by vertical convection and again followed by global diffusion at that level. Determinations of the ionic diffusion coefficients involve great difficulties.

Problems of divergence or flows cannot be treated without considering the effect of the terrestrial magnetic field upon the charged particles. However, it is probable that the ionized components do not control air movements at least to altitudes of the F_2 regions.

Following the termination of the sessions, very interesting colored movies obtained during a rocket flight were shown by Dr. O'Day. The bluish scattered sunlight in the lower atmosphere was clearly visible as was the curvature of the earth. Dr. Menzel also showed the violent motions in the solar corona taken with the coronagraph at Climax Observatory, Colorado.

That the Colloquium was a tremendous success may be attributed only to the wholehearted spirit of cooperation and interest in research shown by the conferees.

(Article contributed to the JOURNAL by N. C. Gerson, Base Directorate for Geophysical Research, AF Cambridge Research Laboratories.)

Climatic Changes.—Is the world warming up?

Scientists who make a career of studying the seasons have been quietly debating this point for the past century. Some believe the weather is staying the same; others maintain that the world's temperature has been slowly rising.

A University of Wisconsin professor has gathered evidence from the way wild animals have been behaving that, in some areas at least, the climate has indeed been changing—and for the warmer.

The scientist is Joseph J. Hickey, professor of wildlife management, who recently reported his findings to fellow biologists holding their annual meeting in Columbus, Ohio. He has drawn his evidence from the changing habits of some half dozen species of mammals and birds that inhabit a belt of which the northern limit runs—or ran—between southern Iowa and New Jersey.

"During the past 30 years we've watched some peculiar things happen to that belt which can only be explained if we assume a climatic change," Hickey says.

The climatic change is not great. It is small enough to permit debate on whether the older weather records are as accurate as those of the present day. But it has—in any case—been great enough to allow some of the animals to extend their breeding range into areas about one hundred miles north of the imaginary southern Iowa–New Jersey line.

"During the years since 1920 various species of animals and birds have more and more frequently been found north of this line," Hickey explains.

One of the bird species, for example, is the cardinal, which has been seen in greater numbers every summer in the area around and to the north of Madison, Wis., since 1920. Other species, including the opossum, turkey vulture, blue wing warbler, and the tufted titmouse, have also been extending summer breeding areas northward.

On the eastern seaboard the whip-poor-will, which is a cold-climate bird, has been retreating northward, apparently before the longer or hotter summers, to be replaced by its southern cousin, the chuck-wills-widow, Hickey states.

The turkey vulture, which is a traditional inhabitant of warm, tropical, and even desert regions, has been observed as far north as upper Minnesota and in 20 years has pushed its range from New Jersey to southwestern Massachusetts.

This same northward movement of animals has been reported by scientists in Europe. In fact, their evidence is in some ways more conclusive than that gathered in the U. S., Professor Hickey says. New bird species from the southern areas of Europe have been invading Greenland, Iceland, Scandinavia, and the Faeroe islands north of Scotland.

"European scientists who have observed this believe firmly that it represents a climatic change," says Professor Hickey. "During the decades since 1910, weather records show two drastic increases in the average winter temperature, totaling more than 15 degrees, which have permitted the migratory bird species to range farther north and occupy new breeding grounds."

The world's glaciers also seem to support the warm-up theory, Hickey points out. Wherever they have been studied it has been found that gradual recession of the ice has been taking place since 1850, with an increasing rate having been observed since 1920. F. E. Matthes of the U. S. geological survey points out that this is an about-face for the glaciers—they had a period of growth during 300 years prior to 1850.

Although changes have taken place in the breeding habits of many American bird species during the past 50 years, all can be explained by some change in the territory into which they've moved—excepting in the case of the half dozen or so species whose movement must apparently be explained by climate, Professor Hickey believes.

The prairie grouse, for example, has moved from southwest Wisconsin northward as far as the Upper Peninsula of Michigan. The prairie horn lark has moved into the Allegheny mountains from its former southern range. But both of these species followed the cutting of forests and the man-made extension of prairie and farm land.

Other species have been moving south. The chestnut warbler and other brush birds have moved into an area in southwest Georgia following destruction of tremendous forests by chestnut blight.

"The movement of all of these species can be accounted for, but we're left without an obvious explanation for the northward movement of the cardinal, opossum, and other species of mammals and birds which at one time had their breeding area limited on the north by the line from Iowa to New Jersey," Professor Hickey points out.

"The climatologists may still be in disagreement with each other regarding the correct interpretation of their temperature records, but our wildlife species are not waiting for their conclusions—they're moving north," he says.